

MULTIPARAMETRIC OPTIMIZATION OF
FOUR-BAR AND SIX-BAR LINKAGES

By

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Abstract of Dissertation Presented to the Graduate Council
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Chairman: Delbert Tesar

Major Department: Mechanical Engineering

An approach for four-bar and six-bar linkage optimization relative to numerous design criteria is presented. A linkage solution set is synthesized through an analytical combination of curvature transform theory, inversion, Robert's cognates and angular cognates under the specification of one of twelve possible types of motion specification. Evaluations are made first on the basis of necessary conditions, and then on the basis of assigned weighting factors and design zones relative to the optimization criteria. Subsequent expansions and refinements through resynthesis about the best linkages are used to approach an optimum solution.

The detailed design for an interactive computer program to implement the analytics is presented and discussed. The

appendices include synthesis procedures, necessary condition evaluations, program module tables and sample programs.


Chairman

CHAPTER I INTRODUCTION

The level of interest in the application of optimization methods to mechanism design seems to be rising exponentially, with the great majority of effort coming from the past decade. The ultimate goals are means to achieve mechanism designs which are globally optimum relative to all pertinent criteria.

Some early attempts to optimize were those of da Vinci (1452-1519) and Newton (1642-1727), who were both involved in specific mechanical design problems, and made modifications of previous designs in order to enhance their performances. Galileo developed a rational mechanical optimization in 1638 by creating an algorithm for the form of a bent beam for uniform strength.

The advent of the differential calculus was fundamental to the development of many modern optimization techniques. The introduction of elegant evaluations of maxima and minima of differentiable functions eliminated the need for discrete evaluations of these functions.

The development of variational techniques is largely credited to Bernoulli, Lagrange and Euler. Chebyshev's [1] involvement in straight-line linkage design led him to develop his well known "Chebyshev polynomials" and optimality

criteria. In 1847, Cauchy [2] contributed the "steepest descent" method.

The German and Russian involvement in mechanism design over the past two centuries produced some extremely sophisticated graphical techniques for design. A graphical approach to four-bar optimization with respect to the transmission angle criterion was presented by Alt [3] in 1925.

The next fundamental influence on optimization was the development of the digital computer. Previously infeasible approaches to optimization became practical and the fields of mechanism design and optimization both changed directions. The birth of modern optimization can probably be credited to the work of Dantzig [4] in the early 1940's, whose work in linear programming included the development of the Simplex method.

In 1948 Svoboda [5] introduced his computational synthesis method of "successive approximations," and in 1954 Levitskii and Shakvazian [6] introduced a method to optimize spatial slider-cranks and crank-rockers on the basis of a set of linear equations from a finite number of precision points.

The work of Freudenstein [7-11] in the latter part of the 1950's formed the basis for modern linkage design procedures. In 1959 Freudenstein and Sandor [10;11] used an IBM 650 and complex number theory to synthesize path-generating mechanisms, and Freudenstein [9] developed a

procedure to optimally approximate a given function using the precision points as design variables.

In the mid 1960's the mechanical design field began to feel the real impact of modern optimization theory, which involved numerical methods initially developed to solve management science and control system problems. Initial application to mechanical designs was primarily in the fields of structural mechanics and aeronautics, but mechanism design was not far behind. The development of such high level languages as Fortran, Algol, APL and Basic have made the use of the computer much simpler, more effective and less system dependent for the engineering community.

A general expression for the linkage optimization problem is:

Minimize Objective Function $F(x)$

where $x = (x_1, x_2 \dots, x_p)$

Subject to:

Inequality constraints $h_k(x) \leq 0; k=1 \text{ to } m$

Equality constraints $e_j(x) = 0; j=1 \text{ to } n$

A condition for the existence of an optimization problem is that the number of design parameters must exceed the number of constraining equations, in order to ensure the existence of "free" parameters.

The objective function may represent the deviation between actual and ideal linkage performance, which may be a measure of prescribed path error ("structural error"), dynamic imbalances, error due to elasticity, or some weighted combination of errors relative to pertinent criteria.

Equality constraints have the effect of narrowing the scope of the optimization problem by reducing the number of free parameters. They commonly take the form of a synthesis step in which a linkage solution set is generated to satisfy prescribed precision points or positions.

Inequality constraints may take the form of design constraints placed on available linkage characteristics, such as link length maxima or minima, range of acceptable transmission angles, or geometric bounds on pivot locations or coupler motions.

The range of possible formulations of the linkage optimization problem is extremely large due to the highly variant sets of 1) linkage applications and 2) optimization criteria. It is highly desirable to organize the available approaches in order to get a feel for what is available, help classify past and present efforts, and perhaps enhance decisions in future efforts. Figure 1-1 shows such a representation.

Esthetically, a closed-form approach is preferred for any problem, but the inherently complex and nonlinear

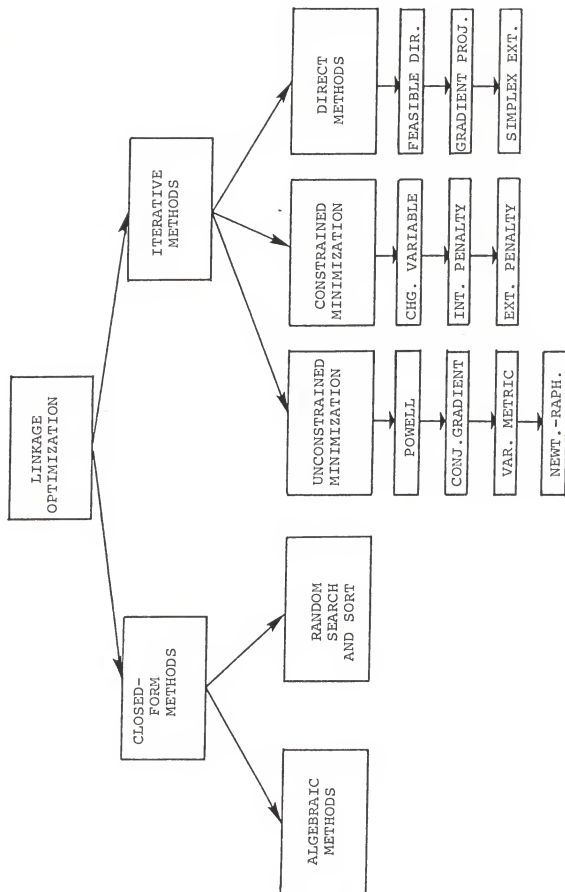


Figure 1-1. Linkage Optimization Methods

linkage problem makes the optimization analytics somewhat difficult to implement. Algebraic approaches, generally limited to small-scale problems, are rarely attempted. Sutherland [12] has developed mixed exact-approximate position synthesis in which some points are matched exactly using Burmester theory, and the objective function contains the least-squares error of approximation of additional precision points. Solutions to the resulting equations can be closed form or iterative depending on the order of magnitude of the problem. Bagci and Lee [13], Bagci [14], and Bhatia and Bagci [15] have developed and expanded on a linear superposition technique in which the error of the loop-closure equations of the linkage system is minimized by partitioning these equations into dyadic loop equations which are linear in terms of the system unknowns.

Random searches are direct and somewhat exhaustive, generally encompassing a synthesis step to generate a large representation of a linkage solution set for a particular problem, and then a rating and sorting step to locate the "best" linkage within this set. As early as 1962, Roth, Sandor and Freudenstein [16] used this approach by generating four-bar linkages satisfying path specifications, and then rating them according to some design criteria (primarily transmission angle). Later Nolle [17] used a random search for function generating linkages. In

1968 Tomas [18] discussed the treatment of linkage synthesis as a nonlinear programming problem, and employed the optimization random-search method of Garrett and Hall [19]. Eschenbach and Tesar [20] generated a large set of linkages computationally and then ranked them according to numerous design criteria, the latter making use of limit design zones and design criterion weights. Two and three function problems are handled under the consideration of ten to twelve criteria acting as sequential filters. Eisenstein and Hall [21] approached the optimization of two-degree-of-freedom function generators by generating a small number of "good" designs, and then generating linkages around these to find a "best" design. Spherical four-bar path generators are handled by Sridher and Torfason [22], who synthesize on four precision points, and then search for the linkage which minimizes the maximum deviation from the prescribed path between the precision points using a random sequential search. The author [23] and the author and Tesar [24] have "grid refinement" technique for four-bars which involves generating two-dimensional discrete representations of the solution set for four coplanar positions, ranking or eliminating these using a weighted design criteria formulation, and then restarting by generating a new "grid" in the most favorable region. Most of the previously mentioned approaches, as well as the iterative methods, can guarantee true global optimization for convex functions only - rarely the case in a linkage problem.

By far most of the emphasis in linkage optimization has been towards the iterative methods. These methods involve "mathematical" (or "numerical") programming techniques which may be (in order of increasing complexity) linear, quadratic, geometric or dynamic. Many techniques exist, Figure 1-1 showing only those used more commonly in linkage applications.

Constrained minimization techniques involve minimization of the objective function in the absence of inequality constraints, and are generally based on a "stepping" iteration, where each "new" step is a function of the location of the "old" step and some formulation of a new direction and step size.

The methods of Powell [25,26] involve unidirectional orthogonal searches to find local minima for each iteration. It is most applicable for problems with non-differentiable objective functions. Suh and Mecklenburg [27] have used Powell's method to operate on matrix-derived objective functions and constraints for spatial mechanisms.

The conjugate gradient technique bears some resemblance to Cauchy's "steepest descent" method but is much more efficient. Unfortunately, convergence degrades as the optima are reached or when the objective function has complicated features or "ridges." Fletcher and Reeves [28] introduced this method in 1964, and later Rees Jones and Rooney [29] employed this method for motion analysis using

an analog computer. Nedić [30] also used an analog simulation, using a combined relaxation and gradient modification of the conjugate gradient approach to circumvent the problem of instability near the optimum. Six-bar function generators were optimized by Chen and Dalsania [31] by applying a least-squares gradient method.

Newton's method in its unmodified form is not often used for linkage problems, largely due to difficulties associated with the selection of initial guesses. One application for this method was found by Rose and Sandor [32], who minimize structural error of four-bar function generators by equalizing error between precision points and at the ends of the specified range. The resulting formulation produces ten nonlinear differential equations to which Newton's method is applied to find a solution.

The so-called "quasi-Newton" method, also called the "variable metric" method, is basically a gradient technique which involves the formulation of a differential matrix which approximates the Hessian as the minimum is reached. Davidson, Fletcher and Powell [33] introduced this method in 1963, and McLaine-Cross [34] used it in 1969 to optimize a crank-rocker four-bar to generate solar declination. This formulation, which involved five nonlinear differential equations, allowed for some error tolerance at the precision points. Optimization of mechanisms with flexible links was discussed by Sevak and

McLarnan [35] by using the variable metric. Conte, George, Mayne and Sadler [36] optimized with respect to dynamic criteria such as shaking force, input torque fluctuation, shaking moment and bearing reactions, using a penalty function in conjunction with the variable metric.

A fourth unconstrained minimization approach, the Newton-Raphson method, is very efficient in terms of convergence, but requires formulation of the Hessian of the objective function at each iteration step. Han [37] used this method in 1966 for general mechanism optimization.

All of the unconstrained minimization formulations except for Powell's method require evaluation of the objective function's differential, but finite difference methods are generally applicable.

The second class of iterative methods are the constrained methods using unconstrained minimization. Generally, this means some modification is made to an unconstrained minimization technique to allow the inclusion of inequality constraints. One approach is a simple change of variable which will inherently disallow violation of specified inequalities, but linkage problems are usually too complex for this approach.

Penalty functions are often used to apply constraints to unconstrained minimization techniques. The interior penalty function introduced by Fiacco and McCormick [38-40] has received quite a lot of attention recently. It is

sometimes called the Sequence of Unconstrained Minimization Technique (SUMT). Fox and Willmert [41] used this technique to optimize a four-bar path-generating linkage using a least-squares error approach. Tranquilla [42] used this method to optimize a four-bar under the geometric coupler-point constraints. Spatial mechanisms were considered by Gupta [43] with constraints on closure, mobility and transmissibility. Bakthavachala and Kimbrell [44] considered four-bar path generator optimization under clearance, tolerance, transmission angle and link size considerations. Conte, George, Mayne and Sadler [36] used the SUMT for optimization with respect to dynamic criteria. Alizade, Mohan Rao and Sandor [45,46] have considered optimization with respect to structural error for two-degree-of-freedom spatial function generators and offset slider-crank path generators. Alizade, Novruzbeikov and Sandor [47] then went on to four-bar function generators with consideration of link length and transmission angle inequality constraints. Kramer and Sandor [48] and Kramer [49] introduced the use of a new type of approximate motion specification in conjunction with an interior penalty function in order to optimally approximate a specified motion. Mariante and Willmert [50] used the interior penalty function to synthesize and optimize a complex convertible top linkage.

Direct methods and linearization methods are used to

deal explicitly with inequality constraints. This group includes the method of feasible directions introduced by Zoutendijk, the gradient projection method, and the extensions of Dantzig's simplex method. Fielding and Zanini [51,52] have discussed the use of modified Simplex methods and considered the optimization of an industrial packaging machine. Also, Youssef, Oldham and Maunder [53] have used the modified Simplex to optimize multi-loop linkages with respect to path error, link length and transmission angle.

The use of variational methods for linkage problems can be traced back to the early work of Freudenstein [9] which later received attention from McLarnan [54], who attempts to minimize structural error by requiring the derivatives at the precision positions to be zero. Huang, Sebesta and Soni [55] and Prasad and Bagci [56] have had more recent applications of variational methods.

Other iterative methods have been introduced by Huang [57], who introduces the concept of sensitivity coefficients, and Lee and Freudenstein [58] and Datseris and Freudenstein [59], who develop the application of Lin's heuristic method (originally developed for the communications field) to mechanism optimization. Moradi and Pappas [60] have developed a "boundary tracking" algorithm for general mathematical programming problems.

Most of the papers discussed so far have dealt primarily with optimization with respect to motion error. If

another criterion is to be considered, it is generally related to transmission angle. Lin [61], Hamid and Soni [62,63], Shoup, Staffer and Weatherford [64], Freudenstein and Primrose [65], Sutherland and Roth [66], Bagci [67], and Savage and Suchora [68] have all dealt with optimization relative to transmission angle. Optimization with respect to dynamic criteria is also a field of interest, with contributions coming from Tomas [69], Benedict and Tesar [70], Berkof and Lowen [71,72], Conte, George, Mayne and Sadler [36], Tepper and Lowen [73], Sadler [74], Elliot and Tesar [75], and Sandler [76]. Optimization of more general mechanical systems such as planetary gear reducers and a shaping mechanism are discussed by Golinski [77], and Osman, Sankar and Dukupati [78] have treated optimization of transmission gears. Optimization using the least-squares method of Gauss has been done by Chi-Yeh [79] and Mansour and Osman [80]. In 1971 Bagci and Prasad [81] and Bagci and Parekh [82] dealt with optimization schemes for coplanar and spherical four-bar and six-bars used for rigid body guidance. Speckhart [83] used minimum weight and the cost criterion as his objective function. Khan, Thornton and Willmert have developed "optimality criterion techniques" for minimizing weight in linkages subject to restrictions in stress and natural frequency [84] and stress and deformation [85]. Rao and Ambekar [86] optimized spherical RRRR function generators, and Sallam and

Lindholm [87] optimized six-bar Watt-1 function generators. Hobson and Torfson [88] introduced a theoretical method which involves analysis of the centrodes of specified motion.

Graphic synthesis and analysis packages have been developed and bear some consideration in any discussion of linkage optimization due to their high level of human/computer interaction, which allows convenient adjustment of representations of "good" designs to achieve "better" designs. Reed and Garrett [89] and Smith and Reed [90] have introduced IMAGE (Interactive Mechanism Analysis through Graphic Exchange) and Ricci [91] has introduced SPACEBAR. Perhaps the most powerful is KINSYN III, introduced by Rubel and Kaufman [92]. All of these employ high-level graphics coupled with linkage synthesis and analysis routines.

The linkage optimization problem has been discussed on a philosophical level by Johnson [93], who discusses stimulants and aids for creativity, such as a building block approach, and systematics of linkages and circuit diagrams. Srivastava and Newcombe [94] introduce a multifactor objective function including such things as pivot location, bearing load limitations, displacement, velocity or acceleration constraints and Grashof constraints. Sutherland and Siddall [95] also use a multifactor objective function with inverse utilics for spherical function

and path generators. Fox [96] describes the state of the art of mathematical programming as applied to the optimization of mechanical components such as springs, shafts and bearings. Oderfeld [97], Starr [98] and Chohan [99] have all presented philosophical discussions of optimization of mechanisms with respect to multiple parameters, with emphasis on ordering of design priorities.

Other literature surveys dealing with mechanism optimization have been presented (in chronological order) by Lindholm [100], Seireg [101] Fox and Gupta [102], Chen [103] and Root and Ragsdell [104].

The "philosophical" papers [93-99] effectively point the way towards the future. A major concern involves methods of achieving globally optimal mechanism designs which accomodate conflicting design requirements. Nearly all of the mechanism optimization schemes to date optimize with respect to one or two parameters. An enormous spectrum of mechanism applications exist, and a large number of criteria by which the overall quality of mechanism solution must be evaluated also exists. To the design engineer, who must be concerned with a mechanism's total functionality, true mechanism optimization must involve a significant number of these parameters.

CHAPTER II
A PHILOSOPHY FOR LINKAGE DESIGN
AND OPTIMIZATION

A. Control of Parameters

In any optimization process, control of the number of free parameters is a primary consideration for linkage problems, where desired motions are specified at a finite number of precision positions, the number of free parameters is determined by the simple relation:

$$p = q - F\ell$$

where p = number of free (optimization) parameters.

q = number of parameters provided by a solution linkage.

ℓ = number of "precision positions" - ℓ .

F = number of functional relations defining each precision position.

which states that the number of free parameters after synthesis equals the number of parameters provided by the solution linkage minus the number of parameters used to satisfy the precision positions. For a given motion

specification type (which defines F), one should create a balance between solution linkage type (which defines q) and number of precision positions (which defines ℓ) in order to describe adequately the desired motion and still leave an acceptable number of free parameters for optimization (between one and three free parameters is desirable).

Naturally there is no guarantee that a design procedure will exist for every combination of q, F and ℓ . For example, one cannot expect to satisfy a large number of precision positions simply by using a sufficiently complex linkage.

In this work twelve types of motion are considered, and for each of these at four specified positions, an acceptable number of free parameters is generated by using either a four-bar linkage or one of the six-bar linkages. The synthesis procedures for the solution linkages are well known and have been collected and organized by Tesar [105].

B. The Optimization Procedure

A synthesis, analysis and resynthesis loop is sufficient as a basis for optimization, and generally allows one to take advantage of the knowledge and insight provided by kinematic synthesis techniques, unlike the more rigid and forceful numerical approaches. The synthesis/

analysis/resynthesis loop to be used for this work is shown in Figure 2-1.

The input to the synthesis step is the precision position description of the desired motion, hereafter termed the motion specification set. A sequence of kinematic synthesis algorithms will produce the output, a solution set of linkages of magnitude ∞ , ∞^2 , or ∞^3 (depending upon q , F and l for the particular problem).

Unfortunately, the analytics used in the synthesis process are not sufficient to guarantee that all (or indeed, any) of the generated linkages will satisfy the specified motion without falling into one or more of a number of physical pitfalls of real linkages, any one of which can make a linkage useless as a solution to the specified problem. For this reason a "necessary condition" step is required. The necessary conditions depend upon the motion specification type, the linkage solution type, and the means by which the linkage is driven. The linkage solution set is mapped onto linear (1-dimensional), rectangular (2-dimensional) or cubic (3-dimensional) spaces, depending upon the magnitude of the solution set space. These representations, termed "grids" due to their discrete, arrayed nature, form the operating environment for all subsequent optimization procedures. Through evaluation of appropriate necessary conditions, portions of the grid representing acceptable linkage solutions are determined and presented to the designer.

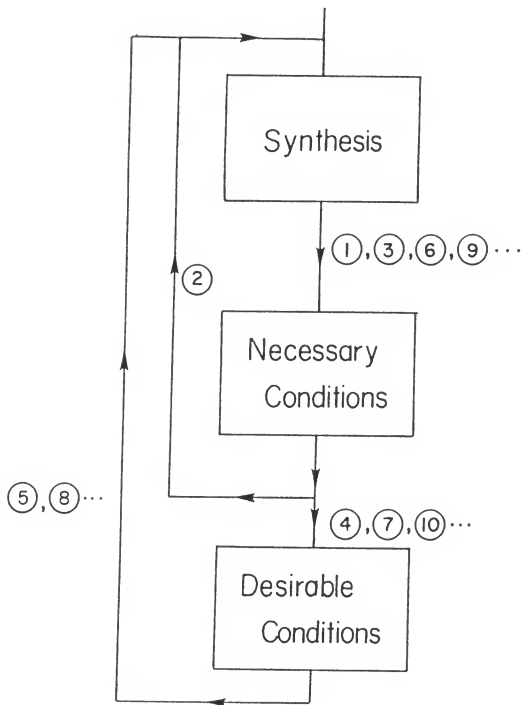


Figure 2-1 The computational steps of a Synthesis/Analysis/Resynthesis Linkage Optimization

The third step, evaluation of "desirable conditions," is where the "quality" of the surviving solution linkages is determined by the user-defined objective function. A desirable condition can be any criterion by which a designer might qualitatively judge a linkage. The grid is passed through these desirable conditions sequentially, and for each one grid "topology" is re-evaluated. This procedure is known as sequential filtering. Some linkages may perform so poorly relative to one or more of these criteria as to be unacceptable, and will therefore be rejected.

The result is a grid of solution linkages in which a portion or portions contain acceptable linkages, with scores indicating relative overall quality.

C. Previous Use of the "Grid" Philosophy

The optimization philosophy employing sequential filtering was first implemented by computer by Eschenbach and Tesar [20] in 1969 for crank-rocker four-bars satisfying specified coplanar motion for four positions. A very large solution set (~62,000 linkages) was passed through the necessary and desirable conditions in a single pass, the optimum linkage possessing the best surviving score. An improvement on this approach was later made by the author [23] and the author and Tesar [24] by introducing the

concept of grid expansion. Grid expansion is a refinement technique by which the three steps of synthesis, necessary condition evaluation and desirable condition evaluation become part of a continuous loop. A region on the grid about the linkages of higher quality is selected, and resynthesis is performed to generate a new grid with a more finely discretized representation (a conceptual magnification) about the selected region. Two or three passes through the loop is generally sufficient.

Due to the typically high attrition of solution linkages in the first pass through the necessary conditions and the high computational expense of the desirable condition analysis relative to the necessary condition analysis, a further improvement can be had by looping back just after the necessary conditions on the first pass (as shown in Figure 2-1). Because desirable conditions are not evaluated initially, a relatively finely discretized grid can be used on this first pass. All of these techniques are employed in the author's thesis program package SOFBAL (Synthesis and Optimization of Four Bar Linkages), which is capable of synthesizing and optimizing all types of four-bar linkages for general coplanar motion.

This successful philosophy is retained for this work, but now twelve types of motion specification are considered. In the next chapter these motion specification types and

the solution linkage types are presented and discussed. Subsequent chapters will treat in detail the analytics behind the steps of synthesis, necessary condition evaluation and desirable condition evaluation.

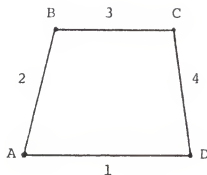
CHAPTER III SOLVABLE PROBLEMS

The previous chapter described an optimization philosophy upon which the efforts of this work are based. In this chapter the range of problems which are solvable using this philosophy (and subsequently developed analytics) is discussed. The range of solvable problems is defined by the three concepts of linkage type, mode of rotational cyclicity and motion specification type.

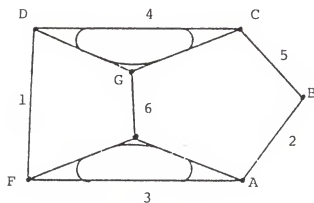
A. Linkage Types

The simplest non-linear mechanism is the four-bar linkage, as shown in Figure 3-1a. It possesses one degree of freedom after one of its links has been grounded, and is conceptually identical in all of its inversions, i.e., regardless of which link is grounded. The four-bar provides eight free parameters.

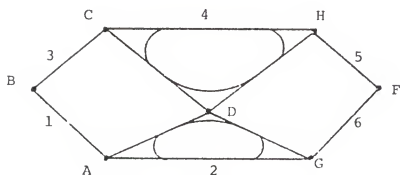
Two types of single-degree of freedom six-bar chain exist, the Stephenson and the Watt, as shown in Figure 3-1b,c. They are both composed of two ternary links and four binary links, and have different conceptual inversions, as shown in Figures 3-2 and 3-3. Each one provides



a. The Four-Bar Chain

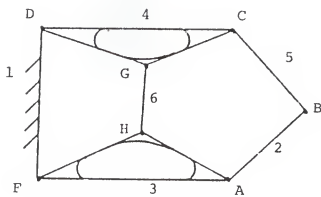


b. The Stephenson Six-Bar Chain

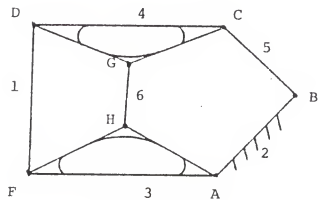


c. The Watt Six-Bar Chain

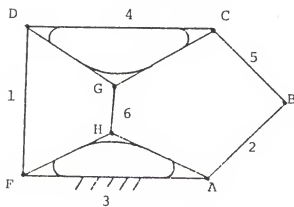
Figure 3-1 Four-Bar and Six-Bar Chains



a. The Stephenson 1

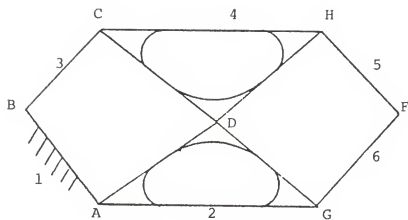


b. The Stephenson 2

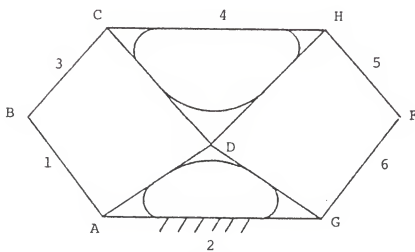


c. The Stephenson 3

Figure 3-2 Inversions of the Stephenson Six-Bar Chain



a. The Watt 1



b. The Watt 2

Figure 3-3 Inversions of the Watt Chain

fourteen free parameters. A "Stephenson 1" is produced if link 1 or 6 of the Stephenson chain is grounded, a "Stephenson 2" if link 2 or 5 is grounded, and a "Stephenson 3" if link 3 or 4 is grounded. A "Watt-1" results from grounding link 1, 3, 5 or 6 of a Watt chain, and a "Watt 2" if link 2 or 4 is grounded. These are available linkage solutions to the problems in this work, and the notation introduced in Figure 3-1 will be adhered to.

Other simple linkages exist, such as the slider-crank and inverted slider-crank four-bars, the geared five-bars and six-bars with sliders and oscillating blocks. They have less general applicability and except for the geared five-bar are simply subsets of the linkages already under consideration.

B. Modes of Rotational Cyclicity

A linkage chain can have a number of distinct types of motion depending upon linkage geometry and the nature of the input drive. These motion characteristic types are hereafter termed "modes of rotational cyclicity."

The possible modes of rotational cyclicity are displayed for the four-bar chain in Figure 3-4, and for the six-bar chains in Figures 3-5 and 3-6. The information in these figures is central to this work. This unfortunate

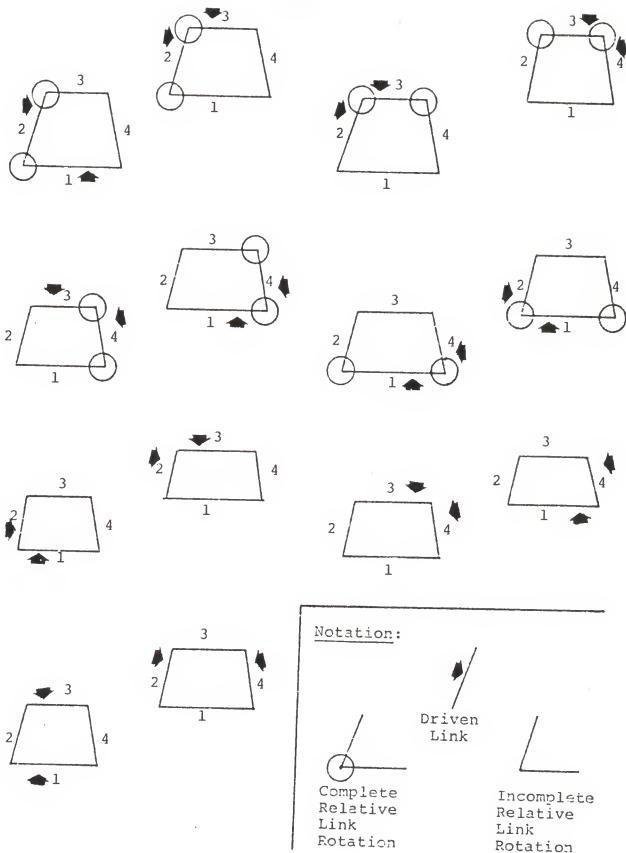
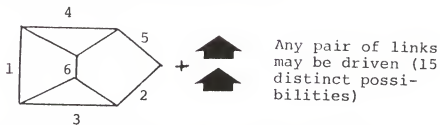


Figure 3-4 Modes of Rotational Cyclicity of the Four-Bar Chain

Stephenson Six-Bars With No Cyclic Links



===== Stephenson Six-Bars With Cyclic Links

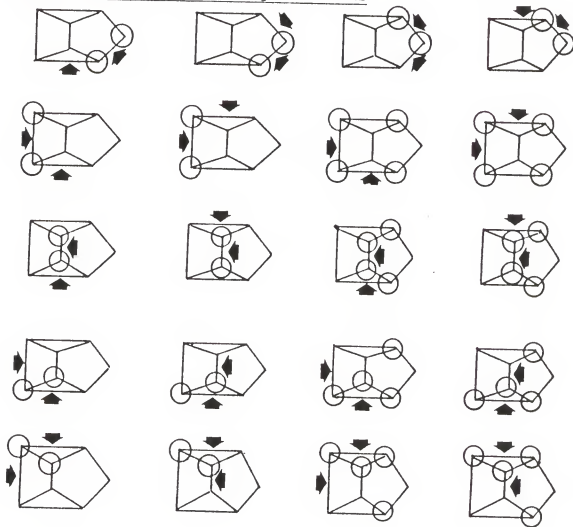
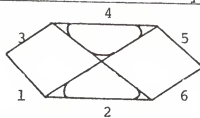


Figure 3-5 Modes of Rotational Cyclicity of the Stephenson Six-Bar Chain

Watt Six-Bars With No Cyclic Links



Any pair of links
may be driven (15
distinct possi-
bilities)

===== Watt Six-Bars With Cyclic Links

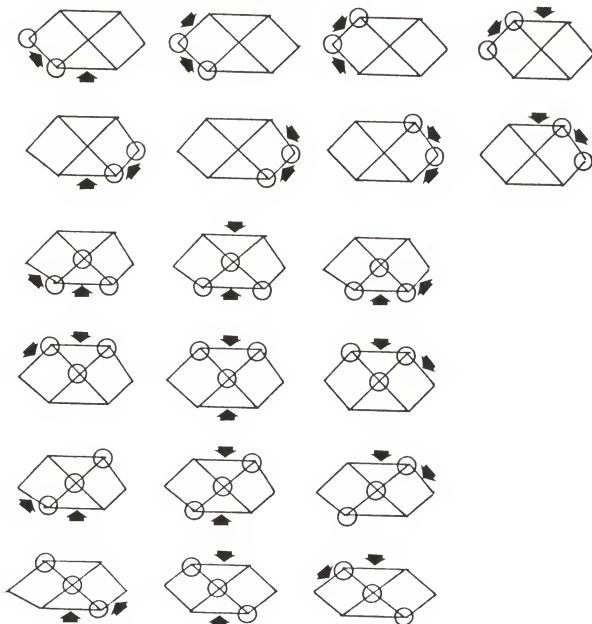


Figure 3-6 Modes of Rotational Cyclicity of the Watt Six-Bar Chain

and exhaustive analysis is necessary because the nature of the necessary condition analysis depends upon the mode of cyclicity. For example, the pivots within a linkage solution at which change point positions (i.e. adjacent link collinearities) can be tolerated depends upon how the linkage is driven.

For our purposes it often becomes useful to consider each chain as free in space with two links driven -- one taking the place of the previously grounded link. In essence, the grounding and driving of links becomes conceptually identical and interchangeable.

At this point a restriction is made in order to eliminate modes of cyclicity for which special knowledge of velocities and system inertias are needed to define motions through change point positions. It will be assumed that when links with two cyclic pivots exist in a chain, one of these links and an adjacent link are driven.

This restriction eliminates from further consideration linkages such as the rocker-driven crank-rocker with complete crank rotation, and the Grashof double-rocker with complete coupler rotation. These linkages must rely upon link momenta or switching of inputs to maintain continuous motion, and are beyond the consideration of this work.

Any given four-bar or six-bar problem will have a subset of the possible modes of cyclicity, depending upon the grounded link and the conceptual equivalence or

non-equivalence of links (a function of motion specification type). As an example, the four-bar solution to the two-link chain problem (Figure 3-7) has seven modes of cyclicity. Modes number 3 and 4 would otherwise be identical except that constraint links 2 and 4 are not conceptually the same due to the nature of the motion specifications: link 2 is contained within the motion specifications, link 4 is not. The same is true for modes 5 and 6. The necessary condition analyses will depend upon 1) required Grashof type for those modes which include cyclic links, and 2) the pivots which are not allowed to pass change points. For example, in mode number 6 a change point is permissible at pivots A, C and D, but not at B, regardless of Grashof type. These considerations are presented in detail in Chapter V.

Figure 3-8 shows some examples of modes of rotational cyclicity which are unacceptable or geometrically impossible. Figures 3-8a and 3-8b might represent the cyclic coupler-driven Grashof double rocker and the cyclic rocker-driven crank-rocker, respectively, which have already been discussed. In Figure 3-8c, control of motion through the change points of pivots r and s is impossible. The motion shown for the six-bar linkages of Figures 3-8d and 3-8f simply cannot exist, and the motion shown for the Watt six-bar linkages of Figure 3-8e would require a linkage with very unusual geometry and an external control of motion for both four-bar chains.

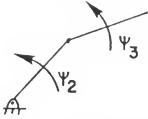
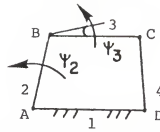
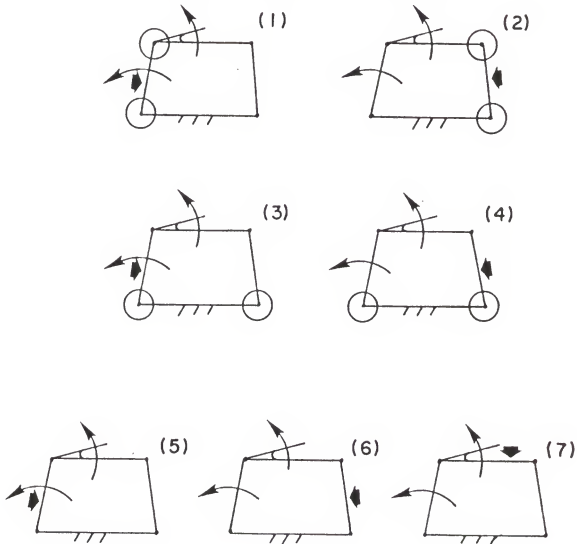
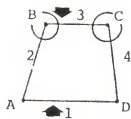
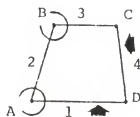
ProblemSolutionApplicable Modes of Rotational Cyclicity

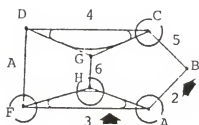
Figure 3-7 Applicable Modes of Rotational Cyclicity of the Four-Bar Solution to the Two-Link Chain Problem



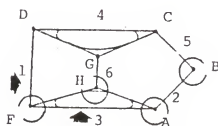
(a)



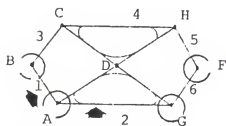
(b)



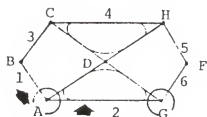
(c)



(d)



(e)



(f)

Figure 3-8 Some Unacceptable Modes of Rotational Cyclicity

C. Motion Specification Types

The types of coplanar problems which are solvable and optimizable using the procedures developed in this work are shown in Figure 3-9. The first and second problems, coplanar path and motion syntheses, are the subject of most linkage optimization efforts.

1) Path Synthesis. The motion of a point is guided by attaching the point to the coupler of an appropriate four-bar linkage.

2) Motion Synthesis. The motion of a body is guided by attaching the body to the coupler of an appropriate four-bar linkage.

3) Angular Coordination of a Two-Link Chain. The angular motion of two connected bodies, one of which is connected to ground, is controlled by attaching these bodies to an appropriate four-bar linkage.

4) Angular Coordination of Two-Cranks. The angular motions of two unconnected bodies, both of which are connected to a common ground, are coordinated by attaching these bodies to an appropriate four-bar linkage.

5) Path-Crank Coordination. The motion of a point is coordinated with the rotation of a body connected to ground by attaching the point and body to an appropriate four-bar linkage.

6) Angular Coordination of Three-Link Chain. The

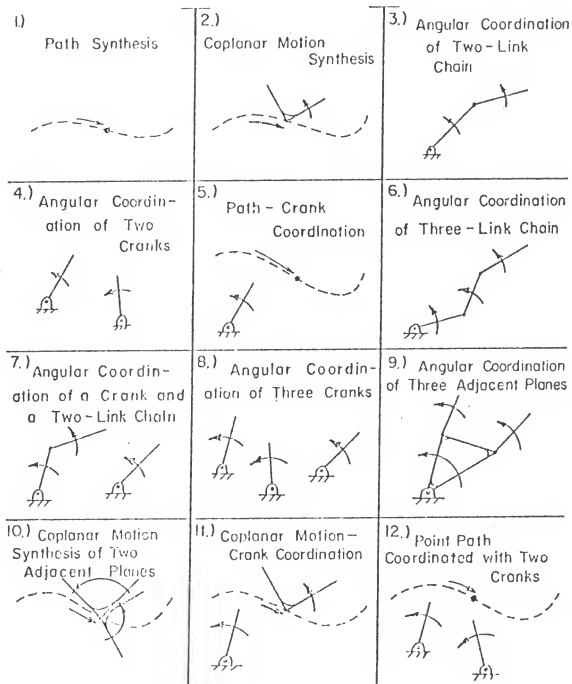


Figure 3-9 Types of Motion Specifications

motion of three bodies connected in a chain (one connected to ground), is guided by attaching these bodies to the links of an appropriate six-bar linkage.

7) Angular Coordination of a Crank and a Two-Link Chain. The motion of two connected bodies (one connected to ground) is coordinated with the rotation of another body which is connected to ground by attaching these bodies to the links of an appropriate six-bar linkage.

8) Angular Coordination of Three-Cranks. The rotation of three non-adjacent bodies, all connected to ground, is coordinated by attaching these bodies to the links of an appropriate six-bar linkage.

9) Angular Coordination of Three Adjacent Planes. The motion of two non-adjacent bodies and a third body which is connected to ground and to which the first two bodies are connected, is controlled by attaching these bodies to the appropriate links of an appropriate six-bar linkage.

10) Coplanar Motion Synthesis of Two Adjacent Planes. The general motion of two connected bodies is controlled by attaching these bodies to the links of an appropriate six-bar linkage.

11) Coplanar Motion - Crank Coordination. The motion of a body is coordinated with the rotation of a body connected to ground by attaching these bodies to the links of an appropriate six-bar linkage.

12) Point Path Coordinated with Two Cranks. The motion of a point and the rotation of two non-adjacent bodies which are connected to a common ground are coordinated by attaching the point and two bodies to the links of an appropriate six-bar linkage.

These problems and their possible linkage solutions are presented in the charts of Appendix A. The steps involved in an actual synthesis procedure are discussed in Chapter IV. Examples of synthesis procedures and necessary condition procedures are presented in Appendix B, and the necessary conditions will be discussed in detail in Chapter V.

CHAPTER IV SYNTHESIS OF LINKAGE SOLUTIONS

In the last chapter the types of problems under consideration were presented (Figure 3-9). The synthesis solution to each consisted of attaching these bodies with specified motions to the links of an "appropriate" linkage. The means by which this "appropriate" linkage is found is the subject of this chapter.

A. The Four Tools of Synthesis

The synthesis procedure for the problems shown in Figure 3-9 depends on the problem type and the solution linkage type. In each case some combination of four synthesis techniques is employed. These techniques are termed the "four tools of synthesis" and are depicted graphically in Figure 4-1.

1.) Synthesis Tool #1. The Curvature Transform

Probably the single most important concept in the field of kinematic synthesis is the curvature transform. All of the synthesis procedures used in this work will use this tool of synthesis at least once to generate constraint links (which may be operated upon by the other tools of

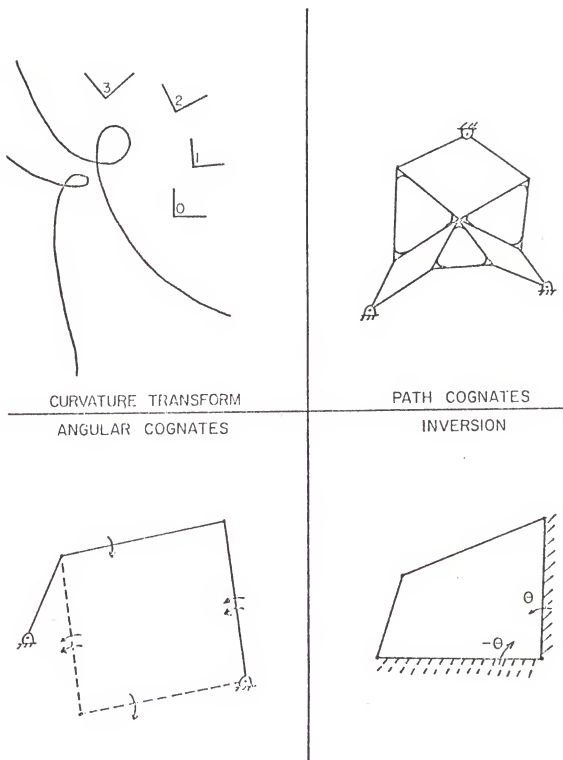


Figure 4-1 The Four Tools of Synthesis

synthesis) in order to produce a functional linkage. For the first three problems the curvature transform alone is sufficient.

Since no linkage provides an infinite number of design parameters the number of functional motion specifications must be finite. If the precision positions are specified in such a way that an independent parameter, such as the coupler angle, undergoes a finite displacement between positions, the specifications are termed finitely separated positions. These are shown for a plane in coplanar motion in Figure 4-2a.

Finitely separated position synthesis of four-bars was first done by Burmester [106] in 1888, using graphical techniques for five positions. In 1964 the problem was solved analytically by Bottema [107] and Primrose, Freudenstein and Sandor [108].

If higher order derivatives of motion at a precision position are specified, they are referred to as infinitesimally separated positions (ISP) as depicted by the conceptual combination of positions in Figure 4-2b. Mueller [109], Allievi [110], Krause [111], Wolford [112] and Veldkamp [113] performed graphical studies of infinitesimally separated positions for five positions.

Mixtures of finitely separated positions and infinitesimally separated positions are termed multiply separated positions (MSP) (Figure 4-2c). The unified theory for MSP was developed by Tesar [114] in 1967.

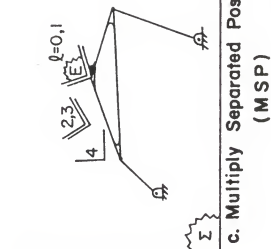
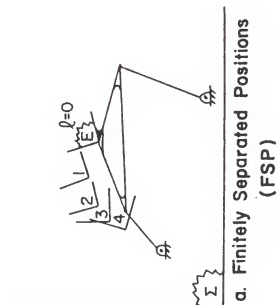
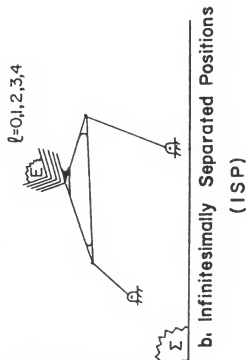


Figure 4-2 Finitely, Infinitesimally and Multiply Separated Positions

All of the previously mentioned work involving ISP and MSP involved geometric derivatives, where the independent parameter is the angular orientation of the body. The specifiable higher-order motions are consequently geometric (slope, curvature, inflection, etc.). In 1974 Myklebust [115] presented a unified theory making dynamic (i.e., time dependent) higher order properties specifiable. This is termed "time state synthesis." For the four-bar problems the dynamic properties are equivalent to the geometric properties, but for a six-bar linkage the use of an equivalent independent parameter for the decoupled subsections of a linkage solution is a necessity. Use of time state synthesis allows true "kinematic synthesis" to be carried out, as conditions can be prescribed for position and its time derivatives (velocity, acceleration, etc.) or combination of inertia and momenta effects, up to the fourth order.

Formulation of the curvature transform

Consider the moving plane in Figure 4-3 where motion is described by the coordinates α , β and γ . Σ is the fixed reference with orthogonal coordinates U and V attached to it, and E is the moving reference with orthogonal coordinates u and v attached. They are related by the "coordinate transform":

$$\begin{aligned} U &= u \cos \gamma - v \sin \gamma + \alpha \\ V &= u \sin \gamma + v \cos \gamma + \beta \end{aligned} \tag{4.1}$$

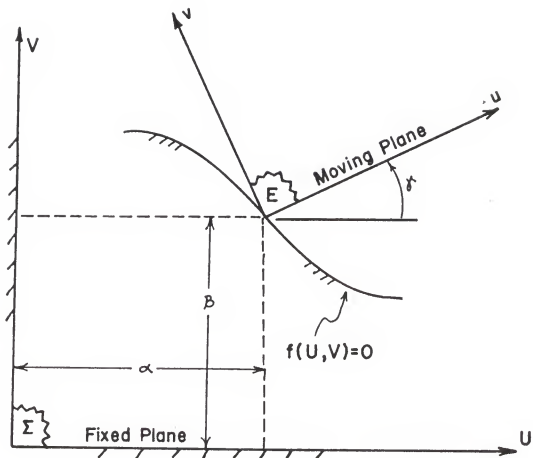


Figure 4-3 Parameters Relating Fixed and Moving Planes

and the "inverse coordinate transform":

$$\begin{aligned} u &= (U-\alpha) \cos \gamma + (V-\beta) \sin \gamma \\ v &= (V-\beta) \cos \gamma - (U-\alpha) \sin \gamma \end{aligned} \quad (4.2)$$

The constrained motion of E can be described in terms of two functional relationships. Letting γ be the independent parameter, let $\alpha = f_1(\gamma)$ and $\beta = f_2(\gamma)$. Links with moving pivots attached to E and fixed pivots attached to Σ might constitute physical constraints. If so, the functional description of the circular motion of a moving pivot (attached at say, point A in E) relative to Σ can take the form $F(U,V) = 0$. Combining this with the coordinate transform (4.1), the constraint equation can be represented by:

$$F = f(u, v, \alpha, \beta, \gamma, Q_0, Q_1, Q_2 \dots) = 0$$

where Q_i 's are constants. For finitely separated specifications of the motion of plane E the above constraint equation takes the form:

$$\begin{aligned} F_j &= f(u, v, \alpha_j, \beta_j, \gamma_j, Q_0, Q_1, Q_2 \dots) = 0 \\ j &= 0, 1, 2 \dots \end{aligned}$$

For infinitesimally separated positions this relationship takes the form:

$$F_k = \frac{d^k}{d\gamma^k} [f(u, v, \alpha, \beta, \gamma, Q_0, Q_1, Q_2 \dots)] = 0$$

For multiply separated positions these two expressions would be mixed. Now, the circular constraint equation can be expressed as:

$$Q_0(U^2+V^2) + 2Q_1U + 2Q_2V + Q_3 = 0$$

For a point A on plane E whose path describes a circle for multiply separated positions, this condition will take the form:

$$g_{\ell}(U,V) = \left. \frac{d^k}{d\gamma^k} [Q_0(U^2+V^2) + 2Q_1U + 2Q_2V + Q_3] \right|_{\gamma=\gamma_{\ell}} = 0 \quad (4.3)$$

If the zeroth position ($g_0=0$) is subtracted, these equations will reduce to:

$$\begin{aligned} G_{\ell}(U,V) = g_{\ell} - g_0 = \left. \frac{d^k}{d\gamma^k} [Q_0(U^2+V^2 - U_0^2 - V_0^2) + 2Q_1(U - U_0) \right. \\ \left. + 2Q_2(V - V_0)] \right|_{\gamma=\gamma_{\ell}} = 0 \end{aligned} \quad (4.4)$$

for $\ell = 1, 2, 3$

and coefficient Q_3 is eliminated.

Now, if the specifications are normalized so that $\alpha_0 = \beta_0 = \gamma_0 = 0$, then $U_0=u$ and $V_0=v$. Substituting the coordinate transform into (4.4):

$$U^2+V^2-u^2-v^2 = \alpha^2+\beta^2+u(2\alpha\cos\gamma+2\beta\sin\gamma)+v(2\beta\cos\gamma-2\alpha\sin\gamma)$$

and the generalized circular constraint function (divided by 2) takes the form:

$$G_{\ell} = \frac{d^k}{d\gamma^k} \left[Q_0 \left(\frac{\alpha^2 + \beta^2}{2} \right) + u(\alpha \cos \gamma + \beta \sin \gamma) + u(-\alpha \sin \gamma + \beta \cos \gamma) \right. \\ \left. + Q_1 (u(\cos \gamma - 1) - u \sin \gamma + \alpha) + Q_2 (u \sin \gamma \right. \\ \left. + u(\cos \gamma - 1) + \beta) \right] \Big|_{\gamma = \gamma_{\ell}} = 0$$

which may be written as:

$$G = Q_0 (A_{0\ell} + uA_{1\ell} + vA_{2\ell}) + Q_{\ell} (uA_{3\ell} + vA_{4\ell} + A_{5\ell}) \\ + Q_2 (uA_{4\ell} + vA_{3\ell} + A_{6\ell}) = 0 \quad (4.5) \\ \ell = 1, 2 \dots$$

where $A_{m\ell}$ coefficients are listed in Table 4-1. They are a function of the motion specifications only.

Now, let:

$$\bar{D}_{\ell} = A_{0\ell} + uA_{1\ell} + vA_{2\ell}$$

$$\bar{E}_{\ell} = uA_{3\ell} - vA_{4\ell} + A_{5\ell}$$

$$\bar{F}_{\ell} = uA_{4\ell} + vA_{3\ell} + A_{6\ell}$$

and the constraint equations now take the form:

$$G_{\ell} = Q_0 \bar{D}_{\ell} + Q_{\ell} \bar{E}_{\ell} + Q_2 \bar{F}_{\ell} = 0 \quad (4.6)$$

For 4MSP motion specifications ($\ell = 1, 2, 3$):

$$Q_0 \bar{D}_1 + Q_1 \bar{E}_1 + Q_2 \bar{F}_1 = 0$$

$$Q_0 \bar{D}_2 + Q_1 \bar{E}_2 + Q_2 \bar{F}_2 = 0$$

$$Q_0 \bar{D}_3 + Q_1 \bar{E}_3 + Q_2 \bar{F}_3 = 0$$

The matrix

$$\begin{bmatrix} \bar{D}_1 & \bar{E}_1 & \bar{F}_1 \\ \bar{D}_2 & \bar{E}_2 & \bar{F}_2 \\ \bar{D}_3 & \bar{E}_3 & \bar{F}_3 \end{bmatrix} \quad (4.7)$$

must be singular if the constraint equations are to be meaningful since these equations are linear and homogeneous with respect to Q_0 , Q_1 and Q_2 .

If this matrix (4.7) is expanded, the result is a cubic in terms of E coordinates u and v . This is the "circle point cubic" and represents the locus of permissible moving pivot locations on E.

The formulation for time state specification synthesis is quite similar. Equation (4.3) will now take the form:

$$G_\ell = \left. \frac{d^k}{dt^k} [Q_0 (u^2 + v^2) + 2Q_1 u + 2Q_2 v + Q_3] \right|_{t=t_\ell} = 0 \quad (4.8)$$

for $\ell = 0, 1, 2$

and the $A_{m\ell}$'s in equation 4.5 will be replaced by time-state motion coefficients termed $D_{m\ell}$'s. The $D_{m\ell}$'s are listed in Table 4-2.

In time-state synthesis γ becomes a dependent parameter, but the $D_{m\ell}$'s reduce to $A_{m\ell}$'s by replacing independent parameter t by γ so that:

$$t \rightarrow \gamma; \quad \frac{d\gamma}{dt} \rightarrow \frac{d\gamma}{d\gamma} = 1; \quad \frac{d^k \gamma}{dt^k} \rightarrow \frac{d^k \gamma}{d\gamma^k} = 0 \quad k > 1$$

and the time-state synthesis formulation reduces to the geometric synthesis formulation.

2.) Synthesis Tool #2. Inversion

The inversion concept essentially involves a change of reference. Two basic types of inversion are of interest. In the first, the motions of planes N and M (Figure 4-4) are defined relative to a fixed plane R, the reference is to be changed to plane N, and the motion of plane M relative to plane N is to be defined (formulations #1, 2 and 3, below). In the second type of inversion the motion of N is defined relative to fixed plane R, the motion of plane M is defined relative to plane N, and the motion of M relative to R is to be defined (formulations #4, 5 and 6, below).

Motion specifications for the two planes which have some ISP content must share a common independent parameter,

or have a defined relationship between independent parameters. The types of motion specification sets which correspond to the formulations that follow are displayed in Table 4-3.

a.) Inversion formulation number 1.

Given: Motion of N relative to R; M relative to R.

Find: Motion of M relative to N (for R_0 reference).

Independent parameter for given MSP: time t.

Independent parameter for derived MSP: time t.

$$\alpha_{mj}^{(k)n} = S_k \cos C + T_k \sin C + \alpha_{n0}^{(k)r}$$

$$\beta_{mj}^{(k)n} = -S_k \sin C + T_k \cos C + \beta_{n0}^{(k)r}$$

$$\gamma_{mj}^{(k)n} = \gamma_{mj}^{(k)r} - \gamma_{nj}^{(k)r} + \gamma_{n0}^{(k)r}$$

Where: j = FSP counter,

k = ISP counter (for example, for k=2, $\alpha^{(k)} = \alpha^{**}$).

$$S_0 = A = \alpha_{mj}^r - \alpha_{nj}^r$$

$$T_0 = B = \beta_{mj}^r - \beta_{nj}^r$$

$$C = \gamma_{nj}^r - \gamma_{n0}^r$$

$$S_1 = \dot{A} + \dot{B}\dot{C}$$

$$T_1 = -\dot{A}\dot{C} + \dot{B}$$

$$S_2 = -2\ddot{A}\dot{C} - \ddot{A}\dot{C} + \ddot{B} - \dot{B}\dot{C}^2$$

$$T_2 = \ddot{A} - \dot{A}\dot{C}^2 + 2\ddot{B}\dot{C} + \dot{B}\ddot{C}$$

$$S_3 = \ddot{\ddot{A}} - 3\ddot{A}\ddot{\dot{C}} - 3\dot{A}\ddot{\dot{C}}^2 + 3\ddot{\dot{B}}\dot{C} + 3\dot{B}\ddot{\dot{C}} - \dot{B}\dot{C}^3 + \dot{B}\ddot{\dot{C}}$$

$$T_3 = \ddot{\ddot{B}} - 3\ddot{\dot{B}}\ddot{\dot{C}} - 3\dot{B}\ddot{\dot{C}}^2 - 3\ddot{\dot{A}}\dot{C} - 3\dot{A}\ddot{\dot{C}} + \dot{A}\dot{C}^3 - \dot{A}\ddot{\dot{C}}$$

b.) Inversion formulation number 2.

Given: Motions of N relative to R; M relative to R.

Find: Motion of M relative to N (for R_0 reference).

Independent parameter for given MSP: γ_n^r

Independent parameter for derived MSP: γ_m^n

This formulation is identical to that of (2a), except that the dots in the S_k and T_k formulations should be replaced with primes ('), and now A' , A'' , A''' , B' , B'' , B''' and C' , C'' , and C''' must be defined.

Let $\alpha_n \doteq \alpha_n^r$; $\alpha'_n \doteq \frac{d\alpha_n^r}{d\gamma_n^r}$; etc. (same for β_n and γ_n).

$\alpha_m \doteq \alpha_m^r$; $\alpha'_m \doteq \frac{d\alpha_m^r}{d\gamma_m^r}$; etc. (same for β_m and γ_m).

Now,

$$A' = \frac{\alpha'_m - \alpha'_n}{\gamma'_m - 1}$$

$$A'' = \frac{\alpha''_m - \alpha''_n}{(\gamma'_m - 1)^2} - \frac{(\alpha'_m - \alpha'_n)\gamma''_m}{(\gamma'_m - 1)^3}$$

$$A''' = \frac{\left\{ (\alpha'''_m - \alpha'''_n)(\gamma'_m - 1)^2 + (\alpha''_m - \alpha''_n)(3\gamma''_m(\gamma'_m - 1)) \right.}{(\gamma'_m - 1)^5}$$

$$\left. - (\alpha'_m - \alpha'_n)(3\gamma''^2_m + \gamma'''_m(\gamma'_m - 1)) \right\}$$

B' , B'' , B''' are similar (substitute β 's for α 's).

$$C' = \frac{1}{\gamma'_m - 1}$$

$$C'' = \frac{-\gamma''_m}{(\gamma'_m - 1)^3}$$

$$C''' = \frac{-\gamma_m''(\gamma_m' - 1) + 3\gamma_m''^2}{(\gamma_m' - 1)^5}$$

c.) Inversion formulation number 3.

Given: Motions of N relative to R; M relative to R.

Find: Motion of M relative to N (for R_0 reference).

Independent parameter for given MSP: γ_n^r, γ_m^r (must be related).

Independent parameter for derived MSP: γ_m^n

Again, this formulation is identical to that of (2a), except that the dots in the S_k and T_k formulations should be replaced with primes ('), and now $A', A'', A''', B', B'', B'''$, C', C'' and C''' must be defined.

Let $\alpha_n \doteq \alpha_n^r$; $\alpha_n' \doteq \frac{d\alpha_n^r}{d\gamma_n^r}$; etc. (same for β_n and γ_n).

$\alpha_m \doteq \alpha_m$; $\alpha_m' \doteq \frac{d\alpha_m^r}{d\gamma_m^r}$; etc. (same for β_m and γ_m).

$\gamma_{nm}' \doteq \frac{d\gamma_n^r}{d\gamma_m^r}$; $\gamma_{nm}'' \doteq \frac{d^2\gamma_n^r}{d\gamma_m^r{}^2}$; etc.

The formulations for A', A'', A''', B', B'' and B''' are equivalent to those of section (2b), under the above notation, except that $\gamma_m \rightarrow \gamma_{nm}$, $\gamma_m' \rightarrow \gamma_{nm}'$, etc., and C', C'' and C''' must

be redefined.

Now,

$$C' = \frac{\gamma'_{nm}}{1 - \gamma'_{nm}}$$

$$C'' = \frac{\gamma''_{nm}}{(1 - \gamma'_{nm})^3}$$

$$C''' = \frac{\gamma'''_{nm}(1 - \gamma'_{nm}) + 3\gamma''_{nm}^2}{(1 - \gamma'_{nm})^2}$$

d.) Inversion formulation number 4.

Given: Motions of N relative to R, M relative to N.

Find: Motion of M relative to R.

Independent parameter for given MSP: time t.

Independent parameter for derived MSP: time t.

$$\alpha_{mj}^{(k)r} = S_k \cos C - T_k \sin C + \alpha_{nj}^{(k)r}$$

$$\beta_{mj}^{(k)r} = S_k \sin C + T_k \cos C + \beta_{nj}^{(k)r}$$

$$\gamma_{mj}^{(k)r} = \gamma_{mj}^{(k)n} + \gamma_{nj}^{(k)r}$$

Where,

$$S_0 = \alpha_{mj}^n$$

$$C = \gamma_{nj}^r$$

$$T_0 = \beta_{mj}^n$$

$$S_1 = \dot{\alpha}_{mj}^n + \beta_{mj}^n \dot{\gamma}_{nj}^r$$

$$T_1 = \alpha_{mj}^n \dot{\gamma}_{nj}^r + \dot{\beta}_{mj}^n$$

$$S_2 = \ddot{\alpha}_{mj}^n - 2\dot{\beta}_{mj}^n \dot{\gamma}_{nj}^r - \beta_{mj}^n \ddot{\gamma}_{nj}^r - \alpha_{mj}^n \dot{\gamma}_{nj}^{r^2}$$

$$T_2 = 2\dot{\alpha}_{mj}^n \dot{\gamma}_{nj}^r + \alpha_{mj}^n \ddot{\gamma}_{nj}^r - \beta_{mj}^n \dot{\gamma}_{nj}^{r^2} + \dot{\beta}_{mj}^n$$

$$S_3 = \ddot{\alpha}_{mj}^n - 3\ddot{\beta}_{mj}^n \dot{\gamma}_{nj}^r - 3\dot{\beta}_{mj}^n \ddot{\gamma}_{nj}^r - \beta_{mj}^n \dddot{\gamma}_{nj}^r - 3\dot{\alpha}_{mj}^n \dot{\gamma}_{nj}^{r^2} \\ - 3\alpha_{mj}^n \ddot{\gamma}_{nj}^r \dot{\gamma}_{nj}^r + \beta_{mj}^n \dot{\gamma}_{nj}^{r^3}$$

$$T_3 = 3\ddot{\alpha}_{mj}^n \dot{\gamma}_{nj}^r - 3\dot{\beta}_{mj}^n \dot{\gamma}_{nj}^{r^2} - 3\beta_{mj}^n \ddot{\gamma}_{nj}^r \dot{\gamma}_{nj}^r + 3\alpha_{mj}^n \ddot{\gamma}_{nj}^r \\ - \alpha_{mj}^n \dot{\gamma}_{nj}^{r^3} + \alpha_{mj}^n \dddot{\gamma}_{nj}^r + \ddot{\beta}_{mj}^n$$

e.) Inversion formulation number 5.

Given: Motions of N relative to R, M relative to N.

Find: Motion of M relative to R.

Independent parameter for given MSP: γ_n^r

Independent parameter for derived MSP: γ_m^r

This formulation is identical to that of (2d), except that the dots in the S_k and T_k formulations should be replaced with primes (').

Let $\alpha_n \doteq \alpha_n^r$; $\alpha_n' \doteq \frac{d\alpha_n^r}{d\gamma_n^r}$; etc. (same for β_n and γ_n)

$\alpha_m \doteq \alpha_m^n$; $\alpha_m' \doteq \frac{d\alpha_m^n}{d\gamma_n^r}$; etc. (same for β_m and γ_m)

Now,

$$\alpha_m' = \frac{d\alpha_m^n}{d\gamma_m^r} = \frac{\alpha_m'}{1 + \gamma_m'}$$

$$\alpha_m'' = \frac{\alpha_m''(1 + \gamma_m') - \alpha_m' \gamma_m'}{(1 + \gamma_m')^3}$$

$$\alpha_m''' = \frac{\left\{ \alpha_m'''(1 + 2\gamma_m' + \gamma_m'^2) - \alpha_m''(\gamma_m' + 2\gamma_m'' + 2\gamma_m' \gamma_m'' + \gamma_m'^2) \right.}{(1 + \gamma_m')^5}$$

$$\left. + \alpha_m' \gamma_m''(2\gamma_m' - 1) \right\}$$

β_m^n , β_m^n and β_m^n are similar (substitute β 's for α 's).

$$\gamma_n^r = \frac{1}{1 + \gamma_m'}$$

$$\gamma_n^{rr} = \frac{-\gamma_m''}{(1 + \gamma_m')^3}$$

$$\gamma_n^{rrr} = \frac{3\gamma_m''^2 - \gamma_m'''(1 + \gamma_m')}{(1 + \gamma_m')^5}$$

f.) Inversion formulation number 6.

Given: Motions of N relative to R, M relative to N.

Find: Motion of M relative to R.

Independent parameters for given MSP: γ_n^r, γ_m^n (must be related).

Independent parameter for derived MSP: γ_m^r

This formulation is identical to that of (2d), except that the dots in the S_k and T_k formulations should be replaced with primes (').

$$\text{Let } \alpha_n \doteq \alpha_n^r ; \alpha_n' \doteq \frac{d\alpha_n^r}{d\gamma_n^r} ; \text{etc. (same for } \beta_n \text{ and } \gamma_n)$$

$$\alpha_m \doteq \alpha_m^n ; \alpha_m' \doteq \frac{d\alpha_m^n}{d\gamma_m^n} ; \text{etc. (same for } \beta_m \text{ and } \gamma_m)$$

$$\gamma_{mn}' = \frac{d\gamma_n^r}{d\gamma_m^n} ; \gamma_{mn}'' = \frac{d^2\gamma_n^r}{d\gamma_m^n^2} ; \text{etc.}$$

The formulations for α_m^n , β_m^n and their derivatives are equivalent to those of section (2c), under the above notation, except that $\gamma_m \rightarrow \gamma_{mn}$, $\gamma'_m \rightarrow \gamma'_{mn}$, etc.

Now,

$$\gamma_n^r = \frac{\gamma_{mn}'}{1 + \gamma_{mn}'}$$

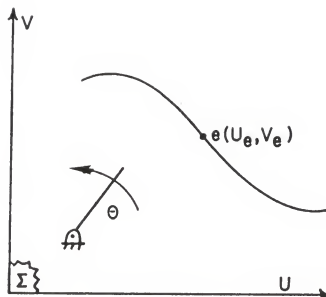
$$\gamma_n^r = \frac{\gamma_{mn}''}{(1 + \gamma_{mn}')^3}$$

$$\gamma_n^r = \frac{\gamma_{mn}'''(1 + \gamma_{mn}') - 3\gamma_{mn}''^2}{(1 + \gamma_{mn}')^5}$$

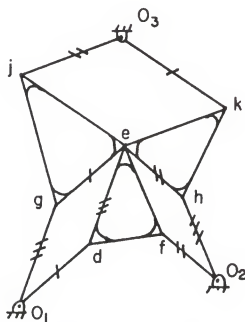
3.) Synthesis Tool #3. Path Cognates

The Roberts- Chebyshev theorem states that three different coplanar four-bar mechanisms will trace identical coupler-curves. An example is shown in Figure 4-5b, where coupler point e has the same motion whether carried by four-bar O_1dfO_2 , four-bar O_1gjO_3 or four-bar O_2hkO_3 . An additional property is that the angular parameters of those links with identical hash marks in Figure 4-5b are identical for any reachable position or position derivative of point e.

These properties can be used to solve the problem of



a. Prescribed Path, and Input Crank



b. Robert's Cognates

Figure 4-5 Use of Path Cognates to Solve the Path-Crank Coordination Problem

path-crank coordination (Figure 4-5a). The synthesis procedure is: 1) specify the motion of coupler def by using the translation of point e and the rotation θ , use the curvature transform to synthesize four-bar O_1dfO_2 , and find its path cognate O_1gjo_3 or O_2hko_3 . Either cognate will coordinate the rotation of a crank parameter θ (attached to link O_1d or O_2h) with the motion of point e (attached to coupler link gej or hek, respectively).

4.) Synthesis Tool #4. Angular Cognates

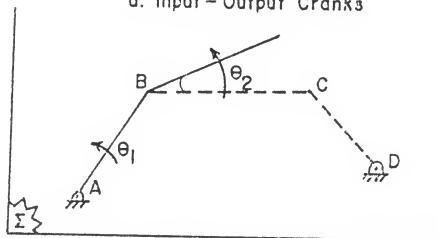
Consider the parallelogram formed by links BCD and BC'D in Figure 4-6c. Regardless of the motion of point B (guided by crank AB for this example) the angular parameters of links BC and C'D are identical. So too for links BC' and CD. Figure 4-6 outlines the use of the concept of angular cognates to solve a crank-coordination problem. The second angular parameter θ_2 is used to define the motion of coupler plane BC, the curvature transform is used to find a second constraint link CD, and the angular cognate of dyad BCD is taken to provide solution linkage ABC'D.

B. Coordination of the Tools of Synthesis

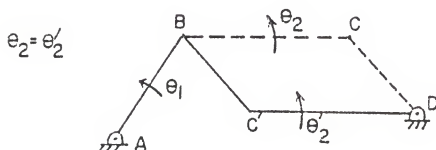
Appendix A contains tables of linkage solutions and recommended synthesis procedures for each of the twelve problem types. These synthesis procedures consist entirely



a. Input - Output Cranks



b. Curvature Transform



c. Angular Cognates

Figure 4-6 Use of Angular Cognates to Solve the Function Generation Problem

of appropriate combinations of the four tools of synthesis just presented, and are generally not unique for any given problem type and linkage solution.

A Stephenson 2 solution to problem number twelve is shown in Figure 4-7. The synthesis procedure utilizes the tools of inversion, path cognates, and (as always) the curvature transform. The procedure begins with a combination of the translational motion specifications of point e and the rotational motion specifications of crank K to create a new motion specification set describing general coplanar motion (Figure 4-7a). Operating on these specifications with the curvature transform will produce constraint links as shown in Figure 4-7b (actually a set of constraint links is produced). Operating on this preliminary linkage solution with path cognate analytics will produce constraint links 7' and 5. These constraint links will move the coupler, link 4, in a way such that coupler point e will move on its specified path in coordination with the specified angular motion of crank link 5. Now, the motion specifications exist for links 4', 5 and 3 relative to fixed link 2. Changing the reference to link 3, it is possible through inversion analytics to define a new motion specification set for plane 4 relative to 3 (Figure 4-7d). Operating on these motion specifications with the curvature transform produces constraint links 1 and 6, with the circle point cubic defining pivots D and G and the

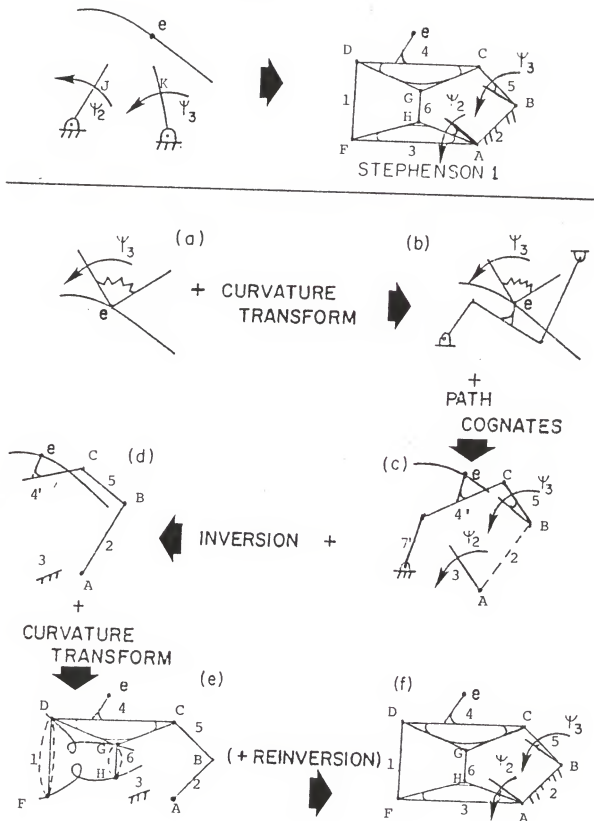


Figure 4-7 A Stephenson-2 Solution to Point/Path Coordination with Two Cranks

center point cubic defining pivots F and H. A conceptual reinversion (which requires no computational activity) produces a Stephenson 2 six-bar linkage which satisfies the motion specifications.

C. Grid Dimension

If 4MSP motion specifications are used, every operation involving a curvature transform generates an infinity of constraint links. If one constraint link is to be selected from this set, a one dimensional grid is required to discretely represent the solution set. Likewise, if two constraint links are to be selected from the set, the grid will have two dimensions. A curvature transform followed by a path cognate operation will generate (2) two-dimensional grids, since each four-bar (which occupies a place on a two-dimensional grid) has two path cognates, a "left" cognate and a "right" cognate.

The example discussed in the last section produces two (2) two-dimensional grids representing link 5 and 7' and another two-dimensional grid representing links 1 and 6. The total, if these grids are combined, is two (2) four-dimensional grids. This problem lies at one extreme of the problems considered in this work. The opposite extreme occurs for problem #'s 3 and 4, whose entire linkage solution sets can be represented by one-dimensional grids. The nature of the component grids is a

function of problem type and linkage solution, but the shape of the overall grid is a function of problem type alone.

D. Dependent Syntheses

The previous example demonstrates an undesirable property which will be hereafter termed "dependent syntheses." Essentially, this property exists whenever a synthesis procedure involves two (or more) curvature transform operations in such a way that the order in which they are performed cannot be switched. In the previous example, the second curvature transform uses the motion specifications of plane 4, which does not exist until plane 4 has been generated by (among other things) the first curvature transform. Thus, every plane 4 generated by the first synthesis will have a unique motion specification set for the second synthesis to operate upon.

This property greatly complicates subsequent analysis and optimization steps, and at this point these difficulties are circumvented by performing a complete optimization on the results of one portion of a dependent synthesis before proceeding to the second synthesis (see Appendix D, problem 12).

Fortunately, not all six-bar synthesis procedures involve dependent syntheses. In fact, all of the problems

except number twelve can be solved by at least one linkage solution without dependent syntheses. These solutions must be considered to be the more desirable from a computational standpoint, and are indicated in Appendix A with the † symbol.

Table 4-1. Motion Coefficients, $A_{m\ell}$

k	$A_{0\ell}$	$A_{1\ell}$	$A_{2\ell}$	$A_{3\ell}$	$A_{4\ell}$	$A_{5\ell}$	$A_{6\ell}$
0	$\frac{1}{2}(\alpha_j^2 + \beta_j^2)$	$\alpha_j \cos \gamma_j$ $+\beta_j \sin \gamma_j$	$-\alpha_j \sin \gamma_j$ $+\beta_j \cos \gamma_j$	$\cos \gamma_j^{-1}$	$\sin \gamma_j$	α_j	β_j
1	$\alpha_j \alpha_j' + \beta_j \beta_j'$	$(\alpha_j' + \beta_j) \cos \gamma_j$ $+(\beta_j' - \alpha_j') \sin \gamma_j$	$(\beta_j' - \alpha_j) \cos \gamma_j$ $-(\alpha_j' + \beta_j) \sin \gamma_j$	$-\sin \gamma_j$	$\cos \gamma_j$	α_j'	β_j'
2	$(\alpha_0')^2 + (\beta_0')^2$	$\alpha_0'' + \beta_0'$	$\beta_0'' - 2\alpha_0'$	-1	0	α_0''	β_0''
3	$3\alpha_0' \alpha_0 + 3\beta_0' \beta_0''$	$\alpha_0'' - 3\alpha_0' + 3\beta_0''$	$\beta_0'' - 3\beta_0' - 3\alpha_0''$	0	-1	α_0''	β_0''

For $k \geq 2$, the position $j = 0$, is assumed as the initial position. When $\ell = j = 0$, $\alpha_0 = \beta_0 = \gamma_0 = 0$. The primes denote derivatives with respect to γ .

Table 4-2. Motion Coefficients, D_{m2}

k	D_0	D_1	D_2	D_3	D_4	D_5	D_6
0	$\frac{1}{2}[(\alpha^2 + \beta^2) - (\alpha_0^2 + \beta_0^2)]$	$(\alpha_j \cos \gamma_j + \beta_j \sin \gamma_j)$ $-(\alpha_0 \cos \gamma_0 + \beta_0 \sin \gamma_0)$	$(\beta_j \cos \gamma_j - \alpha_j \sin \gamma_j)$ $-(\beta_0 \cos \gamma_0 - \alpha_0 \sin \gamma_0)$	$\cos \gamma_j - \cos \gamma_0$	$\sin \gamma_j - \sin \gamma_0$	$\alpha_j - \alpha_0$	$\beta_j - \beta_0$
1	$\alpha_0 + \beta_0$	$(-\alpha_0 + \gamma) \sin \gamma$ $+(\alpha + \beta \gamma) \cos \gamma$	$(-\alpha - \beta \gamma) \sin \gamma$ $+(-\alpha \gamma + \beta) \cos \gamma$	$-\gamma \sin \gamma$	$\gamma \cos \gamma$	α	β
2	$\frac{1}{2}(\alpha_0^2 + \beta_0^2 + \beta_0^2)$	$(-\alpha \gamma - 2\alpha \gamma - 3\gamma^2 + \beta) \sin \gamma$ $+(\alpha \gamma^2 + \alpha + \beta \gamma + 2\beta \gamma) \cos \gamma$	$(\alpha \gamma^2 - \alpha - \beta \gamma - 2\beta \gamma) \sin \gamma$ $+(-\alpha \gamma - 2\alpha \gamma - \beta \gamma^2 + \beta) \cos \gamma$	$-\gamma \sin \gamma$ $-\gamma^2 \cos \gamma$	$-\gamma^2 \sin \gamma$ $+\gamma \cos \gamma$	α	β
3	$3\alpha + (\alpha + 3\beta + \beta^2)$	$(\alpha \gamma^3 - \alpha \gamma - 3\alpha \gamma - 3\beta \gamma - 3\beta \gamma^2 + \beta) \sin \gamma$ $+(-3\alpha \gamma \gamma - 3\alpha \gamma^2 + 3 - 5\gamma^3$ $+ 6\gamma + 3\alpha + 3\beta) \cos \gamma$	$(3\alpha \gamma \gamma + 3\alpha \gamma^2 - \alpha + \beta \gamma^3)$ $-(\beta \gamma - 3\beta \gamma - 3\beta \gamma) \sin \gamma$ $+(\alpha \gamma^3 - \alpha - 3\alpha \gamma - 3\alpha \gamma$ $- 3\beta \gamma \gamma - 3\beta \gamma^2 + \beta) \cos \gamma$	$(\gamma^3 - \gamma) \sin \gamma$ $+(-3\gamma \gamma) \cos \gamma$	$(-3\gamma \gamma) \sin \gamma$ $+(-\gamma^3 + \gamma) \cos \gamma$	α	β

Table 4-3

Related Motion Specification Sets for Two Planes M and N (2ISP)

(Notation: Figure 4-4)

<u>Inversion Formulation #1</u> (Independent parameter - t) <u>Plane N:</u> <u>Plane M:</u> $\alpha_n^r, \beta_n^r, \gamma_n^r$ $\alpha_m^r, \beta_m^r, \gamma_m^r$ $\frac{d\alpha_n^r}{dt}, \frac{d\beta_n^r}{dt}, \frac{d\gamma_n^r}{dt}$ $\frac{d\alpha_m^r}{dt}, \frac{d\beta_m^r}{dt}, \frac{d\gamma_m^r}{dt}$	<u>Inversion Formulation #4</u> (Independent parameter - t) <u>Plane N:</u> <u>Plane M:</u> $\alpha_n^r, \beta_n^r, \gamma_n^r$ $\alpha_m^n, \beta_m^n, \gamma_m^n$ $\frac{d\alpha_n^r}{dt}, \frac{d\beta_n^r}{dt}, \frac{d\gamma_n^r}{dt}$ $\frac{d\alpha_m^n}{dt}, \frac{d\beta_m^n}{dt}, \frac{d\gamma_m^n}{dt}$
<u>Inversion Formulation #2</u> (Independent parameter - coupler angle γ_n^r) <u>Plane N:</u> <u>Plane M:</u> $\alpha_n^r, \beta_n^r, \gamma_n^r$ $\alpha_m^r, \beta_m^r, \gamma_m^r$ $\frac{d\alpha_n^r}{d\gamma_n^r}, \frac{d\beta_n^r}{d\gamma_n^r}, 1$ $\frac{d\alpha_m^r}{d\gamma_n^r}, \frac{d\beta_m^r}{d\gamma_n^r}, \frac{d\gamma_m^r}{d\gamma_n^r}$	<u>Inversion Formulation #5</u> (Independent parameter - coupler angle γ_n^r) <u>Plane N:</u> <u>Plane M:</u> $\alpha_n^r, \beta_n^r, \gamma_n^r$ $\alpha_m^n, \beta_m^n, \gamma_m^n$ $\frac{d\alpha_n^r}{d\gamma_n^r}, \frac{d\beta_n^r}{d\gamma_n^r}, 1$ $\frac{d\alpha_m^n}{d\gamma_n^r}, \frac{d\beta_m^n}{d\gamma_n^r}, \frac{d\gamma_m^n}{d\gamma_n^r}$
<u>Inversion Formulation #3</u> (Independent parameters - coupler angles γ_n^r, γ_m^r (related)) <u>Plane N:</u> <u>Plane M:</u> <u>Other</u> $\alpha_n^r, \beta_n^r, \gamma_n^r$ $\alpha_m^r, \beta_m^r, \gamma_m^r$ - $\frac{d\alpha_n^r}{d\gamma_n^r}, \frac{d\beta_n^r}{d\gamma_n^r}, 1$ $\frac{d\alpha_m^r}{d\gamma_m^r}, \frac{d\beta_m^r}{d\gamma_m^r}, 1$ $\frac{d\gamma_n^r}{d\gamma_m^r}$	<u>Inversion Formulation #6</u> (Independent parameters - coupler angles γ_n^r, γ_m^r (related)) <u>Plane N:</u> <u>Plane M:</u> <u>Other</u> $\alpha_n^r, \beta_n^r, \gamma_n^r$ $\alpha_m^n, \beta_m^n, \gamma_m^n$ - $\frac{d\alpha_n^r}{d\gamma_n^r}, \frac{d\beta_n^r}{d\gamma_n^r}, 1$ $\frac{d\alpha_m^n}{d\gamma_m^n}, \frac{d\beta_m^n}{d\gamma_m^n}, 1$ $\frac{d\gamma_n^r}{d\gamma_m^n}$

CHAPTER V

THE NECESSARY CONDITIONS

Linkages generated through the use of the kinematic synthesis techniques of Chapter IV are definitely capable of satisfying the specified positions. Unfortunately, the analytics presented there take no consideration of linkage performance in positions other than those specified. Real linkages may fall into a number of physical pitfalls, any one of which is sufficient to eliminate the linkage as a viable solution to the specified motion problem. The analytics now presented are for "necessary conditions" which must be satisfied before a given solution linkage can be considered for further optimization analysis. The conditions which apply to a given problem are dependent upon the solution linkage type, the means by which the linkage is to be driven and the nature of its motion (i.e., mode of cyclicity). It is primarily because of the dependence of the necessary conditions upon mode of cyclicity that the concept of the latter has been developed.

A. Four-Bar Necessary Conditions

1.) Order

It is entirely possible that a linkage solution will

satisfy the specified positions in an improper sequence, as displayed in Figure 5-1 for a four-bar linkage. Six possible orders exist for 4FSP:

0 1 2 3	0 1 3 2	0 2 1 3
0 3 2 1	0 2 3 1	0 3 1 2

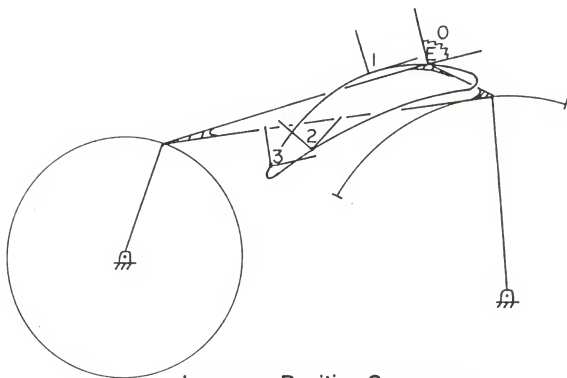
Only those on the left would be acceptable, assuming the linkages can be driven in either direction.

Fortunately, relatively few linkages are eliminated by order considerations. First of all, motion specifications placed in a relatively smooth and continuous sequence tend to be satisfied naturally. Secondly, the specified cases with higher ISP content diminish the order problem. Finally, problems which have motions coordinated with a crank automatically satisfy specified orders.

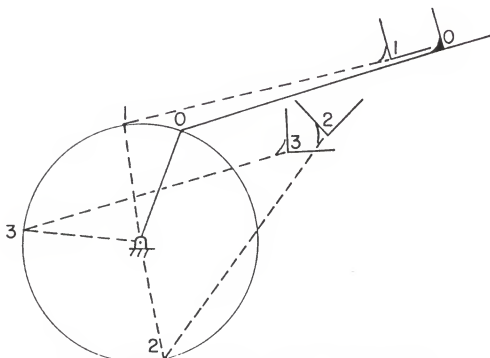
The most direct way to analyze order is to use the method of Modler [116], which divides the circle point cubic into segments which represent constraint links of the same order. These segments are delimited by the image poles and the Ball point. This is a powerful tool for graphical analysis, but for discrete computational analysis it is simpler and more direct to inspect exhaustively the sequence at specified positions of all constraint links.

2.) The Grashof Criterion

A four-bar chain may have either zero pivots with complete rotation, or two adjacent pivots with complete



a. Improper Position Sequence



b. Order Analysis (Crank Rocker)

Figure 5-1 Order

rotation. The latter situation occurs if the linkage satisfies the Grashof condition:

$$l + s < p + q$$

where

l = length of longest link

s = length of shortest link

p, q = length of other two links

Grounded four-bar chains which satisfy this relationship are known as "Grashof four-bars," and take the forms shown in Figure 5-2 a,b,c. They are simply different inversions of one another, and are generally more useful than non-Grashof linkages. The crank-rocker, for example, can be used to generate an arbitrary shape with a coupler point while its crank is driven continuously, and the drag-link is often used for continuous function generation. In a Grashof four-bar only those pivots which are cyclic will ever enter change points.

The non-Grashof four-bars take only one form (regardless of inversion), the non-Grashof double-rocker. This type of linkage tends to dominate a linkage solution grid generated through the curvature transform, but has the unfortunate property of being able to reach "change points" (discussed in next section) at any of its pivots. Control of linkage motion through these change points requires special considerations.

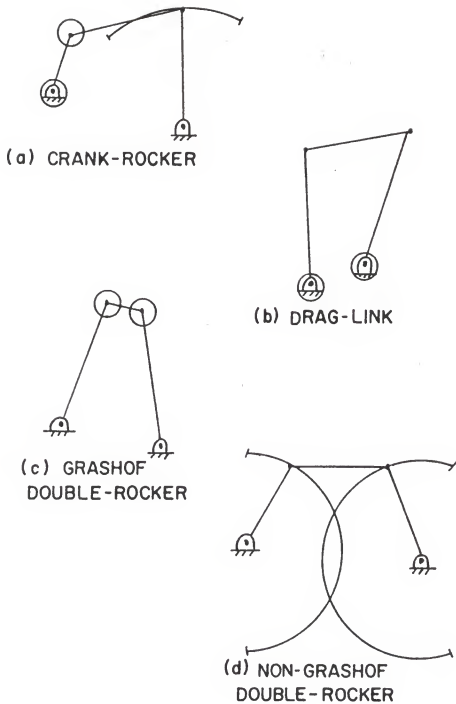


Figure 5-2 Four-Bar Grashof Types

3.) Branching and Change Points

A change point for a given pivot is that position at which attached links become collinear. This position, which might lie between specified positions, may or may not be acceptable depending upon the nature of the linkage and how it is driven.

a.) Dyad change points

In most cases change points in the dyad of a Stephenson or Watt six-bar indicate simultaneous disconnection (or a limit in motion of the rest of the linkage, Figure 5-3b). At this point the dyad could pass from one dyad "type" (relative angular orientation, Figure 5-2c) to the other. One generally wishes to verify that the specified positions are satisfied while the dyad maintains its type, although one exception is that of the cyclic dyad-driven six-bar linkage (Figure 5-3d).

There are two times at which dyad type can be checked. If both links of the dyad are defined by the motion specifications, such as the Stephenson 3 solution to a three-link chain problem (dyad links 2 and 5 are the first two links of the chain), it is a simple matter to simply check the motion specifications before synthesis. More often, however, one of the two links of the dyad is generated in the synthesis procedure, and dyad-type must be checked during necessary condition analysis.

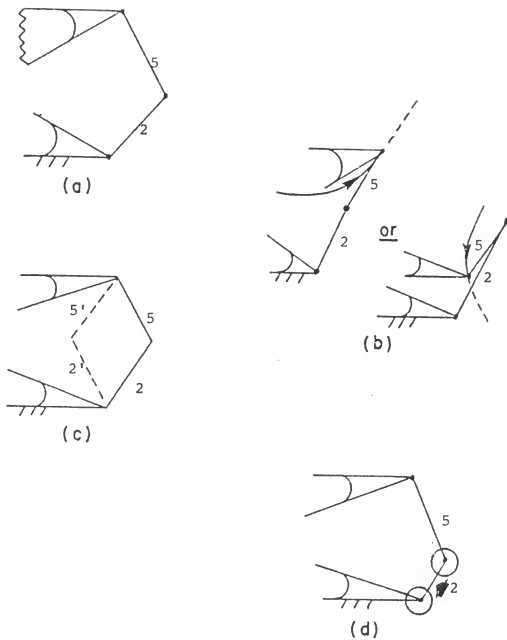


Figure 5-3 Dyad Change Points

b.) Change points in four-bar chains

All of the solution linkages can be considered as single four-bar chains, a combination of four-bar chains and dyads, or a combination of two four-bar chains. A detailed analysis of change points within four-bar chains is therefore very worthwhile. It involves considerations of both synthesis procedure and mode of cyclicity.

If a four-bar chain (or any Grashof type) is driven as shown in Figure 5-4a through a change point at the indicated pivot, the dyad portion of the chain connected to this pivot may go into either type. This is beyond the direct control of the driven members. If specified positions are to be satisfied in a positive manner, such a change point must be avoided. Similarly, four-bar chains driven through opposite links (Figure 5-4b) cannot tolerate change points at any pivots, for the same reason.

It may be assumed that at least one of the links in the chain has been synthesized through use of the curvature transform. Waldron [117, 118] has developed graphical techniques which operate directly on Burmester cubics, identifying sections which represent usable constraint links (for four-bar chains) on the basis of change-point considerations. These techniques are used extensively in the necessary condition analysis of this work, so detailed descriptions now follow.

i.) Determination of "permissible links. Let the name

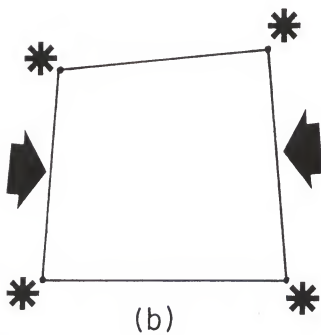
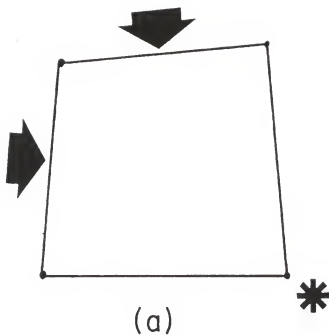


Figure 5-4 Change Point Considerations in a Four-Bar Chain

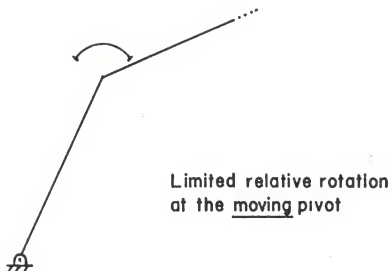
"trailer" signify constraint links with limited rotation at the moving pivot and the name "rocker" signify constraint links with limited rotation at the fixed pivot (Figure 5-5). For example, the follower of a crank-rocker four-bar is both a trailer and a rocker, but a constraint link of a drag-link four bar is a trailer only. Trailers defined by circle- and center-point cubics which have moving pivot rotation less than π for the 4MSP are termed "permissible". The first technique of Waldron is used to determine sections of the circle point cubic which represent permissible constraint links.

Let ψ_{ij} be the rotation of the coupler relative to the constraint link in moving from positions i to j . This angle will be defined between $-\pi$ and π , clockwise positive. Let n denote the position with the smallest or most negative value of ψ_{ij} so that it defines one end of the angular range. If ψ_{nj} is ≥ 0 for all possible values of j , the angular range must be less than π . If it can be established that for some value k , ψ_{jk} is always either positive or negative, the constraint link under consideration must be permissible.

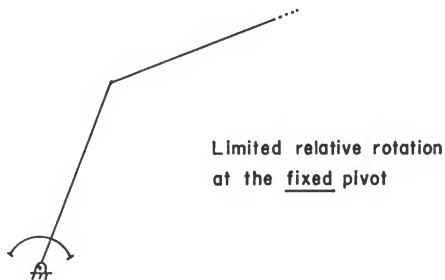
The following property can be used to find the sign of ψ_{jk} :

A line through the image pole P_{ij} and circle point A bisects the angle ψ_{ij} (see Figure 5-6a).

This property applies to any ψ_{ik} , where k is another position, so the angle ψ_{jk} can be found from:

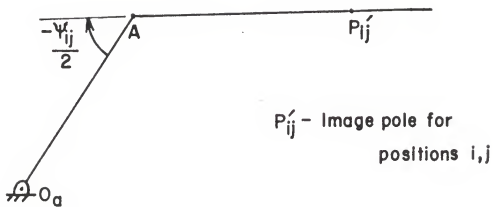


a. "Trailer"

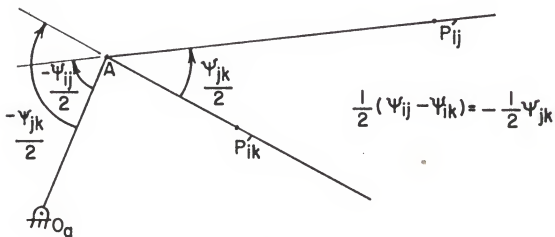


b. "Rocker"

Figure 5-5 Trailer and Rocker



a.



b.

Figure 5-6 Determination of Relative Moving Pivot Rotation for 2FSP

$$\frac{\psi_{jk}}{2} = -\left(\frac{\psi_{ij}}{2} - \frac{\psi_{ik}}{2}\right) \quad (\text{see Figure 5-6b})$$

Now the location of circle points with positive and negative values of the ψ_{jk} in terms of P'_{ij} and P'_{ik} (Figure 5-7c) can be determined by using the property:

The three lines through diametral points and a third point on a circle will define a right triangle.

For circle points lying on the right side of $\overline{P'_{ij}P'_{ik}}$ and outside the circle, $0 < \psi_{jk}/2 < \pi/2$, so that $0 < \psi_{jk} < \pi$, and for circle points on the same side but inside the circle, $\pi/2 < \psi_{jk}/2 < \pi$ (Figure 5-7a). On the opposite side of $\overline{P'_{ij}P'_{ik}}$ the signs will be reversed. Figure 5-7b shows this construction overlayed onto the corresponding circle point curve.

Two values of i could be chosen: the only requirement is that i be distinct from j and k . Interestingly enough, if evaluations were made using both possible values of i some spaces in the plane would have conflicting signs, but the circle point cubic will occupy none of these spaces. Another interesting property is that the signs will not change as the circle point cubic passes through an image pole, but will change as the circle point cubic crosses the circle or line segment singly.

The portions of the circle point cubic which represent permissible constraint links will be defined by those portions for which all the components of any of the strings

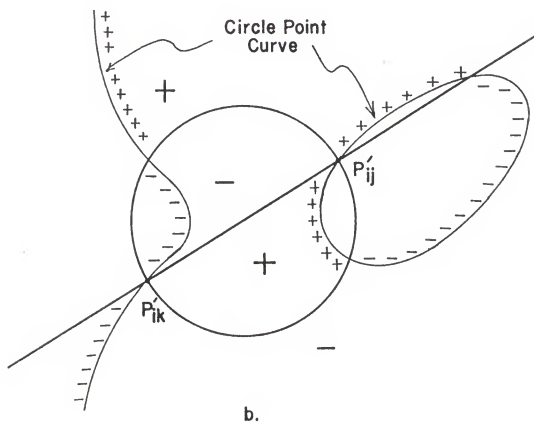
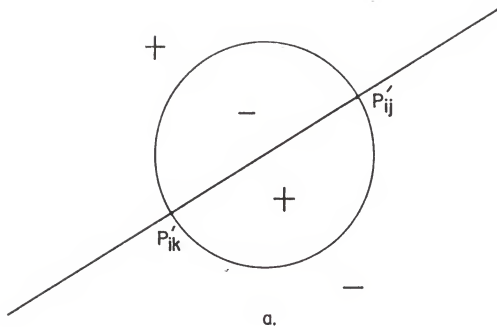


Figure 5-7 Determination of Permissible Trailers for 2FSP

$$\begin{array}{ccc}
 \psi_{01}' & \psi_{02}' & \psi_{03} \\
 \psi_{10}' & \psi_{12}' & \psi_{13} \\
 \psi_{20}' & \psi_{21}' & \psi_{23} \\
 \psi_{30}' & \psi_{31}' & \psi_{33}
 \end{array}$$

have the same sign.

ii.) Determination of "compatible" links. A "compatible" constraint link is one which can be used in combination with a permissible constraint link so that the 4MSP will be satisfied on the same branch. The previous analysis is used to eliminate portions of the cubic which represent constraint links which will definitely pass through a change point at the moving pivot while satisfying the 4MSP (regardless of the other constraint link chosen). A permissible constraint link which has been selected from a portion of the permissible circle point cubic might still pass through a change point if coupled with an "incompatible" constraint link.

Let O_aA be a selected permissible constraint link. Figure 5-8a shows one in the four positions in which the motion specifications are satisfied. ψ_{0j} defines the relative coupler rotation in reaching position j from position 0. Figure 5-9b shows O_aA in the zeroth position with angles $-\psi_{01}$, $-\psi_{02}$ and $-\psi_{03}$ drawn through point A. In this example, $-\psi_{01}$ and $-\psi_{03}$ represent the extremes of the angular range. Now, if a circle point representing a possible second constraint link lies in region Z, it must cross

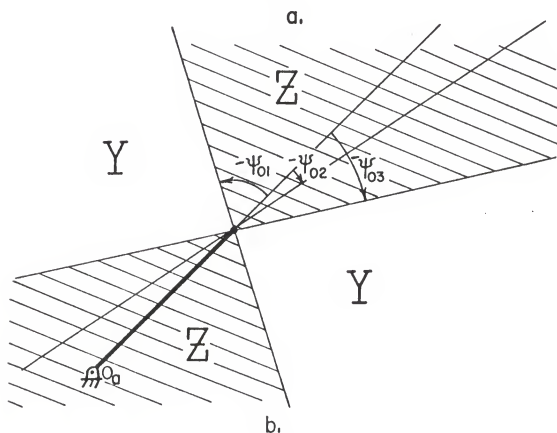
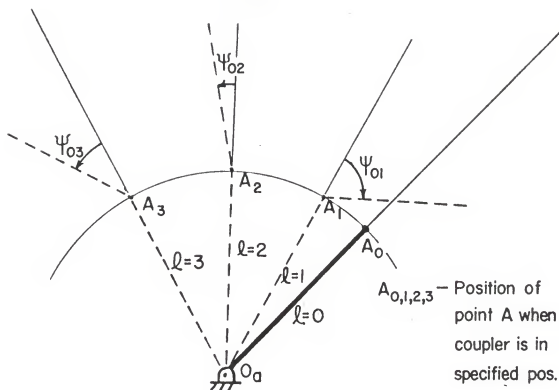


Figure 5-8 Determination of Compatible Constraints

line $\overline{O_a A}$ to reach the extremal positions. As this happens a change point occurs at point A. Therefore, the portions of the circle point cubic which lie in the Z regions represent constraint links which are incompatible with the selected permissible link.

It is quite possible that the circle point cubic will not pass into the compatible regions denoted by Y. If so, another permissible link must be selected. Note that the Y regions disappear altogether for nonpermissible links, having no possibility of finding compatible constraint links.

c.) General branching analyses of four-bar chains in six-bar linkages

Displayed in the left-hand column of Figure 5-9 are the possible forms of four-bar chain which may be needed as all or part (for a six-bar) of a linkage solution. The chains on the right are the different inversions in which the chain on the left may actually be synthesized. It is necessary to treat these inversions as they stand. A number of "necessary condition procedures" have been developed to handle the range of possible synthesis inversions. (The conditions under which order considerations can be ignored have been discussed in part A of this chapter).

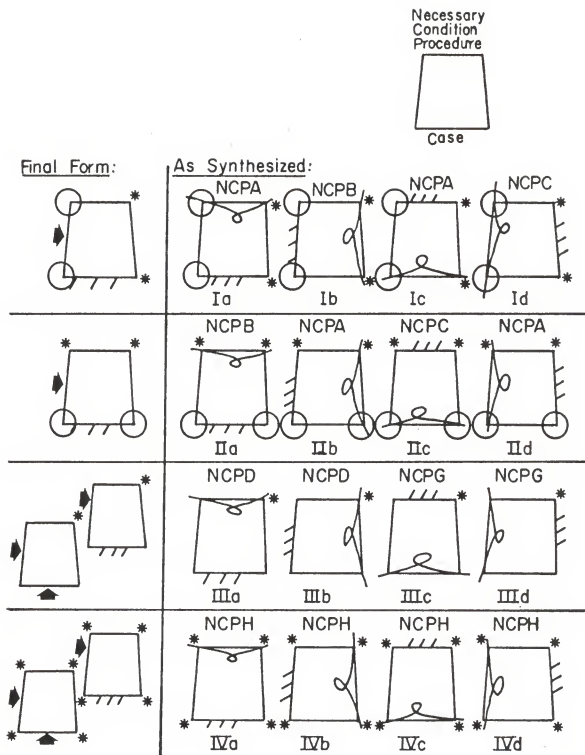


Figure 5-9 Necessary Condition Procedures for Four-Bar Chains

Necessary Condition Procedure A (for cases Ia, Ic, IIb and IId)-

- 1) Choose a follower-constraint-link to be a trailer/rocker.
- 2) Choose a crank-constraint-link of proper order which is compatible with the follower.
- 3) Test Grashof type for a crank-rocker.

Necessary Condition Procedure B (for cases Ib and IIA)-

- 1) Choose a constraint link to be a trailer of proper order.
- 2) Choose another constraint link to be a trailer which is compatible with the first link.
- 3) Test Grashof type for a drag-link.

Necessary Condition Procedure C (for cases Id and IIC)-

- 1) Invert to change reference to the coupler link, i.e. generate the motion specifications of the fixed plane relative to the moving plane, interchange the circle-point and center-point cubic curves, etc.
- 2) Follow procedure B.

Necessary Condition Procedure D (for case IIIA)-

- 1) Choose a follower-constraint-link to be a trailer.
- 2) Choose a crank-constraint-link of proper order which is compatible with the follower.

Necessary Condition Procedure G (for case IIIC)-

- 1) Invert to change reference to the coupler link.
- 2) Follow procedure D.

Necessary Condition Procedure H (for cases IVa,b, c and d)-

- 1) Choose a constraint link which is a trailer of proper order.
- 2) Choose a second constraint link which is a trailer which is compatible with the first link.
- 3) Check mutual compatibility.
- 4) Invert to change reference to the coupler link.
- 5) Repeat 1), 2) and 3) (no need to check order again).

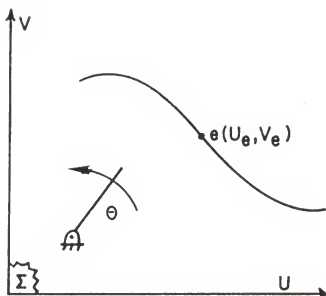
In some cases, tests for order and trailer/rocker are unnecessary if Grashof type are known, but are included because they improve the chances of producing a desired Grashof type. The Grashof type test is relatively expensive computationally because constraint link combinations, not just single constraint links, are under evaluation.

d.) Analysis of branching in path-cognates

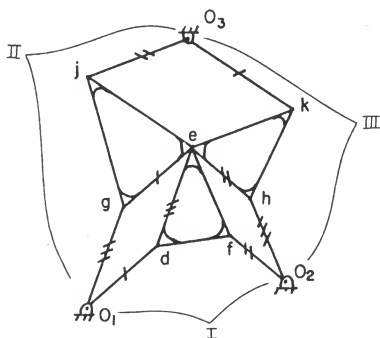
For motion specification types #5 and #12 (Figure 3-9) four-bar chains are produced by performing a curvature transform followed by a generation of path cognate four-bars.

We assume four-bar I in Figure 5-10b has been produced by the curvature transform and its path cognates, four-bars II and III, are to be usable as all or part of a linkage solution. What tests can be made on four-bar I to ensure proper performance relative to change points in four-bars II and III? This question is central to establishing practical necessary condition analyses for path-cognate problems.

A study of four-bar II will now be made in detail. All comments made hereafter regarding "left" cognate four-bar II also apply to "right" cognate four-bar III. The modes of cyclicity which II may take are shown in Figure 5-12. Each of these modes will involve a separate analysis (necessary condition procedures I $\rightarrow$ N) to be performed on four-bar I to ensure proper performance in II.



a. Prescribed Path, and Input Crank



b. Robert's Cognates

Figure 5-10 Use of Path Cognates to Solve the Path-Crank Coordination Problem

Necessary condition procedures $I \rightarrow N$ take advantage of some unusual properties of the "cognate 10-bar" (Figure 5-10b):

i.) A cognate 10-bar will be made up of one of three sets of three four-bars: either three drag-links, three non-Grashof double-rockers, or a Grashof double-rocker and two crank-rockers.

ii.) When O_3 in four-bar II is at a change point, O_2 in four-bar I is also at a change point.

iii.) When g in four-bar II is at a change point, d in four-bar I is also at a change point.

iv.) When O_1 in four-bar II is at a change point, links $\overline{df}$ and $\overline{O_1O_2}$ in four-bar I become parallel.

v.) When j in four-bar II is at a change point, links $\overline{O_1d}$ and $\overline{O_2f}$ in four-bar I become parallel.

Proof of (ii): Referring to Figure 5-13a,

$$\text{let } \lambda = \frac{|\overline{de}|}{|\overline{df}|} = \frac{|\overline{gj}|}{|\overline{ge}|} = \frac{|\overline{O_1O_2}|}{|\overline{O_1O_2}|}$$

Now, from [105;79]

$$\overline{Z}' = \lambda e^{i\alpha}(\overline{Z}) \quad \text{and} \quad \overline{R}' = \lambda e^{i\alpha}(\overline{R})$$

Therefore, when $\overline{Z}$ and $\overline{R}$ coincide, $\overline{Z}'$ and $\overline{R}'$ will also coincide, and change points will occur simultaneously in four-bar I at O_2 and in four-bar II at O_3 .

Proof of (iii): Referring to Figure 5-13b,

$$\alpha + \phi_A + \theta_A = \alpha + \phi_B + \theta_B = 2\pi$$

$\theta_A = \theta_B$ because O_1 ged is a parallelogram,

$\phi_A = \phi_B = 0$ at simultaneous change

points for g in four-bar II and d in four-bar I.

Proof of (iv): Referring to Figure 5-13c, it follows from path-cognate property (ii) that change points occur simultaneously at O_1 in four-bar II and at O_2 in four-bar III. In this position $\overline{O_2h}$ and $\overline{O_2O_3}$ become collinear.

Therefore $\theta_{\overline{O_2h}} + \beta = \theta_{\overline{O_1O_2}}$ and $\theta_{\overline{fe}} + \beta = \theta_{\overline{df}}$

where θ refers to "the angle of ..."

Since O_2 hef is a parallelogram, $\overline{fe}$ and $\overline{O_2h}$ are parallel. It follows that:

$$\theta_{\overline{O_1O_2}} = \theta_{\overline{df}}, \text{ so } \overline{O_1O_2} \parallel \overline{df}$$

Proof of (v): Referring to Figure 5-13d, it follows from path cognate property (iii) that pivot j in four-bar II and pivot k in four-bar III will pass through change points simultaneously. In this position $\overline{O_3j}$ and $\overline{O_3g}$ are collinear, as are $\overline{O_3k}$ and $\overline{O_3h}$. Due to the parallelograms and similar triangles, it follows:

$\overline{ge}$ and $\overline{eh}$ are collinear and

$$\overline{ge} \parallel \overline{O_1d} \text{ and } \overline{eh} \parallel \overline{O_2f}$$

$$\therefore \overline{O_1d} \parallel \overline{O_2f}$$

To use (iv), we wish to ensure that O_1 in four-bar II does not pass through its change point by verifying that link $\overline{df}$ never becomes parallel to link $\overline{O_1O_2}$ in four-bar I while moving in such a way as to satisfy the four specified positions. Every four-bar chain, regardless of Grashof type, has two positions in which two opposite links become parallel. The total motion of the four-bar can be conceptually divided into two segments delimited by these critical positions. The test to be performed, therefore, is a verification that all specified positions lie on the same motion segment.

Since pivots d and f are equidistant from a line through O_1 and O_2 at the critical positions, it follows that throughout one motion segment d is farther away from $\overline{O_1O_2}$ than f , and throughout another motion segment d is closer. If the "altitude" of d is consistently closer or farther from $\overline{O_1O_2}$ than f in all specified positions, then four-bar II will not have a change point at O_1 (and four-bar III will not have a change point at O_2) while satisfying the specified positions.

To make use of (v) similar arguments can be made. If the "altitude" of f (relative to $\overline{O_1d}$) is consistently lesser or greater than the "altitude" of O_2 for all specified positions, pivot j in cognate four-bar II and pivot k in cognate four-bar III will not have change points in moving through the specified positions.

Properties (ii), (iii), (iv) and (v) are represented pictorially by Figure 5-11. With this theory, necessary condition procedures for path cognate problems can be created. Waldron and Stevenson [119] have performed an analysis similar to that used in the first and third of these procedures.

Necessary Condition Procedure I.

Follow necessary condition procedure C for four-bar I (tests for a Grashof double-rocker). Path cognates II and III will be usable crank rockers with cranks at O_1g .

Necessary Condition Procedure J.

Follow necessary condition procedure A for four-bar I (tests for a crank-rocker). If O_2f is the crank in I, four-bar II will be a usable crank-rocker with a crank at O_3j : if O_1d is the crank in I, four-bar III will be a usable crank-rocker with a crank at O_3k .

Necessary Condition Procedure K.

Follow necessary condition procedure B for four-bar I (tests for a drag-link). Path cognates II and III will be usable drag-links.

Necessary Condition Procedure L.

- 1) Perform necessary condition procedure D on four-bar I to indicate change points at d (and g in cognate III).
- 2) Perform necessary condition procedure D on four-bar I to indicate change points at f (and h in cognate III).

Necessary Condition Procedure M.

- 1) Test the distance of f and O_2 relative to O_1d for the specified positions. If the "altitude" of f is consistently lesser or greater, no change points will occur at j in cognate II or k in cognate III.

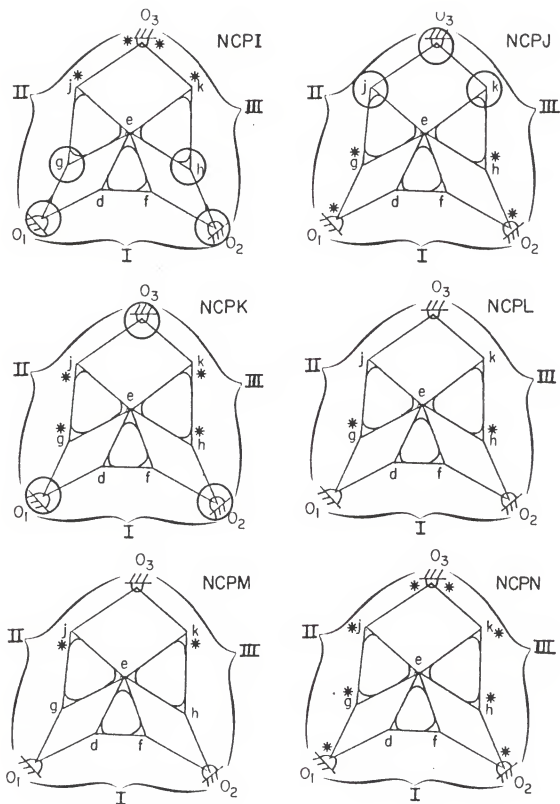
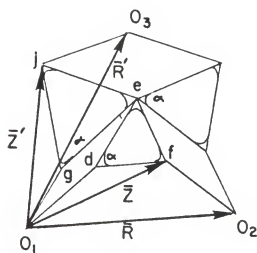
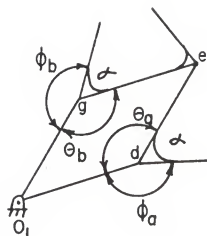


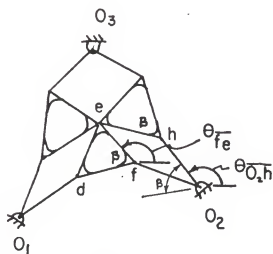
Figure 5-12 Necessary Condition Procedures for Cognate Four-Bar Linkages



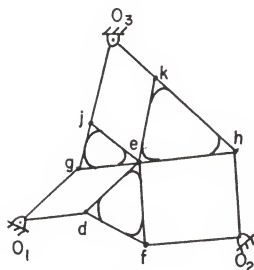
(a)



(b)



(c)



(d)

Figure 5-13 Path Cognate Change Point Derivations

Necessary Condition Procedure N.

- 1) Perform necessary condition procedure L on four-bar I to indicate change points at g in cognate II and h in cognate III.
- 2) Invert coupler and ground, swap constraint links in I, and execute necessary condition procedure L again, to indicate change points at O_3 in cognates II and III.
- 3) Execute necessary condition procedure M on four-bar I to indicate change points at j in cognate II or K in cognate III.
- 4) Test the distance of d and f relative to $\overline{O_1O_2}$ for the specified positions. If the "altitude" of d is always less or greater, no change points will occur at O_1 in cognate II or O_2 in cognate III.

B. Six-bar Necessary Conditions

The necessary conditions presented thus far are sufficient for the necessary condition analysis of four-bar linkages and the constituent parts (i.e., dyads and four-bars) of six-bar linkages. Several conditions involving the combination of these parts into complete six-bar linkages are still needed.

1.) Dyad Connection

The fact that a six-bar linkage is capable of reaching specified positions without change point or order difficulties is not enough to ensure non-disconnection of a linkage member while moving between the specified positions. Conceptually one can imagine a four-bar linkage whose coupler pivot passes outside (or inside) of the bounds of possible motion of dyad pivot q (Figure 5-14a). Verification of this condition is made using a dyad-based analysis of the entire linkage at discrete intervals.

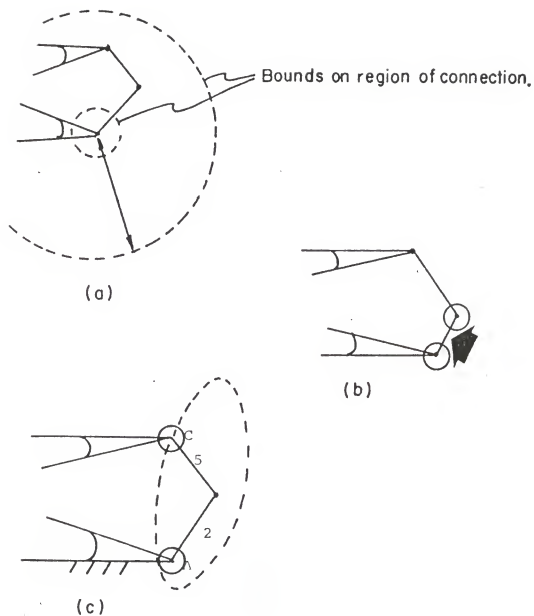


Figure 5-14 Dyad-Related Necessary Conditions

2.) Dyad Disconnection

In contrast to dyad connection, it may be necessary to verify dyad disconnection. This occurs when a Stephenson six-bar linkage is driven by means of a dyad link (Figure 5-14b). Since a dyad analysis cannot be performed on a Stephenson chain with a dyad link as crank, it becomes necessary to drive the dyad through the four-bar through its full range of motion (defined by dyad disconnection) to make sure the dyad link is cyclic. At the same time the four-bar is checked throughout its range of motion for change points. One additional check of relative dyad link length is made to ensure that the appropriate dyad link is rotating completely.

3.) Non-g-Function Reversal

Peculiar to Stephenson 2 six-bar linkages which are driven by one of the two links connected to ground is a necessary condition involving the first order influence coefficient, the g-function. It occurs because the driven link may reach a limit in angular motion which is defined by a higher order geometry constraint. Since it is impossible to analyze a Stephenson 2 directly using a dyad-based analysis, the test is performed by inverting to a Stephenson 3 and driving the linkage at the four-bar chain. If ternary link 3 is the driven member in the original Stephenson 2, the offending position occurs

as the g -function (i.e., relative angular velocity) of link 2 relative to 3 approaches zero and link 2 comes to a halt. On either side of this position $g_{2/3}$ assumes opposite values. If dyad link 5 is the driven member in the original Stephenson 2, a critical position occurs when $g_{5/2}$ approaches zero. In either case, the specified positions must be satisfied without the linkage passing through a critical position.

4.) Encirclement and Non-encirclement

In a Stephenson chain, if the coupler point C encircles point A, as shown in the example of Figure 5-14c, pivots C and A both have complete relative rotation. If encirclement does not occur (and the chain is not cyclicly dyad driven), pivots C and A will have only relative rotation. In either case the dyad will not change type. Either of these motion characteristics (or its exclusion) may be desirable in a particular application.

The necessary conditions connection, disconnection, g -function reversal, encirclement and non-encirclement are evaluated simultaneously during a single dyad-based motion analysis. The range through which the Watt 2 or Stephenson 3 inversion of the six-bar linkage is driven depends upon the mode of cyclicity. If the mode of cyclicity involves no links with complete rotation, the linkage will be driven in small increments from the zeroth to the third specified

position. If the chain is cyclically driven through a dyad link 2 or 5, the linkage is driven through the four-bar chain's full range of possible motion between dyad disconnections. A third possibility occurs if the chain is cyclically driven through a four-bar chain link, in which case the entire linkage is driven through a complete cycle.

CHAPTER VI THE DESIRABLE CONDITIONS

Chapter IV presented the synthesis procedures by which linkage solution sets of one to four dimensions are generated. The necessary conditions, which serve to eliminate from the linkage solution set those linkages with some unacceptable physical characteristics, were treated in Chapter V. Now the desirable conditions, which serve to sort and rank the remaining linkages relative to a user-defined objective function, will be discussed. These conditions constitute the final step in the synthesis/analysis/resynthesis loop by which optimization occurs.

A desirable condition may be any criterion by which the quality of a linkage solution may be judged. The range of application for linkages is enormous, and so too is the range of possible desirable conditions. Nine generally useful desirable conditions have been considered for this work.

A. The Desirable Conditions

1.) Pivot Location

In many applications the designer has restrictions related to moving pivot locations or mounting locations relative to the moving bodies. Depending on the problem type and the linkage solution type selected, some of the

pivots may be prespecified but the location of others will vary throughout the solution set. If some latitude is not available in these locations the designer must reduce his specification. Otherwise he may use the 4MSP motion specifications and treat location of the variable pivots as an optimization criterion.

2.) Link Length Ratio

The link length ratio is simply the length of the longest specified link divided by the length of the shortest specified link (i.e., $LLR \geq 1$) and is indicative of dynamic characteristics of a linkage. Previous optimization schemes dealing with four-bar coplanar motion problems [20,23] have automatically considered all four links in this ratio. For six-bar problems it becomes important to allow the designer to specify the links of particular interest.

3.) Link Size

Kinematic and dynamic characteristics may be function of link sizes relative to overall motion parameters. Again, some of the links may be prespecified, but all problems have variable links.

The link length ratio and link size criteria are both simple geometric linkage parameters which can provide insight into the dynamic characteristics of a linkage. Although they are not particularly reliable, they are useful because they are computationally inexpensive and can

filter out inferior linkages before more expensive desirable conditions are evaluated.

The five following desirable conditions are also geometric linkage parameters which act as indexes of the dynamic characteristics of linkages. They are the result of detailed studies of cam system dynamics by Tesar and Matthew [120] and were recently applied to Geneva mechanisms by Taat [121] and Taat and Tesar [122].

4.) g-function

The g-function is a first order geometric derivative relating output and input motions. It is a reliable indicator of dynamic properties related to output velocities, kinetic energies, force magnifications and input torque requirements. For six-bar problems it often becomes desirable to monitor the maximum values of the g-functions of two output links.

5.) Δh

The h-function is a second order geometric derivative relating output and input motion. It is a good predictor of dynamic properties related to inertia forces and internal deformation. Here the interest is in monitoring peak-to-peak values.

6.) hrms

The root-mean-square of the h-function over a cycle

of operation is an indicator of system precision and internal bearing loads. It is desirable to minimize the overall root-mean-square of the h-function.

7.) Torque

The functional product of the first- and second-order geometric derivatives throughout a cycle (gh) is indicative of the driving torque needed to maintain a constant input. The peak-to-peak value is predictive of speed fluctuation or dynamic windup at the input.

8.) Input Shock Level

A combination of first-, second- and third-order geometric derivatives given by the relationship $\tau \approx gh' + (h)^2$ is an index of shock-level felt at the input of a system. Higher values will be indicative of increasing significance of dynamic properties related to vibration, noise, wear and backlash.

Desirable conditions 3-8 must rely upon a dyad-based analysis, and are evaluated simultaneously for each solution linkage. Higher-order geometric properties are evaluated using a finite-difference differentiation of the zeroth order motion analysis. Taking advantage of the dyad-analysis is a final desirable condition unrelated to system dynamics:

9.) Overall Boundary

This condition is used to rank solution linkages relative to overall space required. Some designs will place "soft" constraints on working volumes in the form of preferred regions, while others may have "hard" absolute restrictions created by other system components.

B. The Objective Function

The linkages which have survived to the desirable conditions are ordered using some user-defined objective function. This function may take the form:

$$N = \sum_k N_k = \sum_k W_k S_k \quad (6.1)$$

$$k = 1, 2, 3 \dots$$

where N represents a cumulative score attached to a particular linkage, k is the desirable condition counter, N_k is a linkage score for condition k , W_k is a weight signifying relative importance of condition k , and S_k is a score signifying performance of the linkage relative to condition k .

The weighting factor W_k is set by the designer, who must make this decision relative to the application using his own experience and judgement. In this work both weights and scores may be assigned within a range of 0 to 10. A weight of 10 signifies a condition of primary

importance and a weight of 0 signifies a condition of no importance. A condition score of 0 signifies ideal performance (almost never attained) and a score of 10 delimits the bound of acceptable performance. In this way a linkage receives a cumulative score on a demerit system: the linkage completing the analysis with the lowest score is the optimum one in the solution set.

There are a number of ways to correlate score S_k and linkage performance. The simplest way is to use a linear scale between ideal performance and the acceptable limit. The score S_k is then generated by:

$$S_k = 10(P_2/P_3) \quad (6.2)$$

where P_2 = difference between ideal
and actual performance.

P_3 = difference between ideal and
unacceptable performance.

An alternative used by Eschenbach and Tesar [20] used ranking zones in which "best", "good" and "acceptable" regions are defined and assigned score values. A better approach was also used in the same paper, where the ratio of actual to maximum value is operated upon by an exponent n assigned by the designer:

$$S_k = \left(\frac{P_2}{P_3} \right)^n \quad (6.3)$$

This approach has the advantage that n can be chosen to assign performance scores in a non-linear fashion within the acceptable region. Note that 6.3 is equivalent to 6.2 when $n=1$ and a factor of 10 is included.

The approach used in this work is the result of a combination of these concepts and was introduced in the author's thesis [23]. The designer is asked to specify for each desirable condition an "ideal", "preferable" and "acceptable" performance, to which scores of 0, 1 and 10 are attached respectively. Any linkage which falls outside of the "acceptable" bounds is discarded. A smooth exponential can be fitted to these performance specifications and linkage scores evaluated on the basis of the exponential:

$$S_k = \left(\frac{P_2}{P_1} \right)^{\left(\frac{1}{\log(P_3/P_1)} \right)}$$

where P_1 = difference between preferable
and ideal performance

P_2 = difference between actual and
ideal performance

P_3 = difference between acceptable
and ideal performance

Figure 6-1 shows scoring as a function of relative placement of ideal, preferable and acceptable regions.

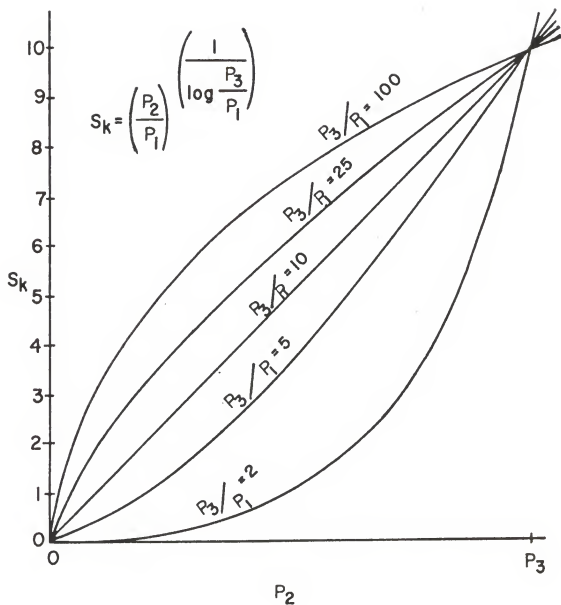


Figure 6-1 Desirable Condition Scoring Curves

One-dimensional desirable conditions (e.g., link lengths) will have single-dimensional regions. Two-dimensional desirable conditions (e.g., pivot locations) will have two-dimensional regions, and these regions may now take the form of circles or rectangles.

CHAPTER VII
PROGRAMMING FOR FOUR-BAR
AND SIX-BAR OPTIMIZATION

A. Philosophy of Program Modularity

The three major computational blocks in the synthesis/analysis/resynthesis approach to linkage optimization and their execution sequence (Figure 7-1) are common to the optimization of all twelve of the motion specification types. The difference between problems lie within these blocks.

To gain an understanding of flexibility needed in an all-encompassing four-bar and six-bar optimization program package, consider that most of the motion specification types can be solved by several types of mechanism, each entailing separate synthesis procedures (fourty-eight combinations of motion specification/solution linkage exist). Every combination has four to fifteen modes of cyclicity, each one requiring some procedure for necessary condition evaluation (the same is true for desirable conditions to a lesser extent).

The program package providing a computational base for the analytics presented in this work is also intended to serve as a foundation for a future mechanisms package

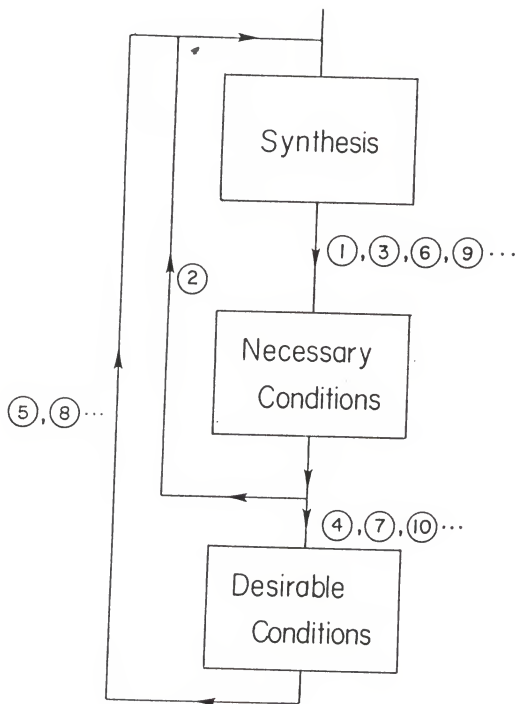


Figure 7-1 The Computational Steps of a Synthesis/Analysis/Resynthesis Linkage Optimization

coordinating linkage synthesis and optimization, cam synthesis and analysis, mechanism shaking force, shaking moment and torque balancing, spring, energy and dashpot synthesis, dynamic analysis and eventually spatial and spherical analytics. Just to meet the goals of synthesis and optimization, however, the package must be capable of handling the analytics for an estimated 500 different problem/linkage/mode of cyclicity combinations.

A most feasible, practical, flexible and marketable approach to the optimization package and its proposed expansion can be based on a programming technique used by Pollock [123] for a dynamic linkage analysis package. The package consists of a pool of modules of equivalent program hierarchy from which a user can select and coordinate those needed for a particular application.

The optimization package has been broken up into such a pool of modules (Table 7-1) consisting of an input and output module for each computational block, five synthesis modules, fourteen necessary condition modules, four desirable condition modules and four general support routines. The function and input/output of each module is presented in Appendix C.

The advantages of the Pollock approach to program structure pertain to both the optimization package and its proposed expansion:

- 1.) A central control program is not necessary. The

package never becomes so large as to become unmanageable.

2.) The user designs his program for his particular application. The only modules needed in his workspace are the ones actually executed. Only pertinent I/O is performed.

3.) The independent nature of the modules permits easy expansions and substitutions, and greatly simplifies maintenance.

B. Control of Input/Output

The projected package could be implemented on almost any interactive system providing sufficient memory, but is very likely most conveniently implemented in an APL environment. All further discussion will relate to an APL-based package.

Data flow between modules utilizes user-designated files, and all user/computer interaction is interactive. Like a Pollock dyad-analysis routine, each module requires file numbers as input. Unlike a dyad-analysis routine (which outputs data into files concatenated onto those already existing), the designer also specifies the locations in which output files are to be placed. Overwriting of obsolete data from the previous optimization loop is now possible and in fact necessary. An example of a typical program for each motion specification type is presented in Appendix D. To write such programs, the user would:

1) Look through Appendix A, find the problem type applying to his problem, and select a solution linkage type.

2) Familiarize himself with the synthesis procedure, select appropriate synthesis modules from Appendix C, and outline the synthesis portion of his program:

```

V PROBLEM
  INITIAL} Input
  SYNINP
  SCRVTNR}
  SPTHCOG} Tools of Synthesis

  SYNOUT } Output

```

3) Select from available modes of cyclicity (Appendix B and Chapter IV), familiarize himself with the necessary condition analysis, and through use of Appendix C write the necessary condition portion of the program:

```

  NECINP} Input
  NCPK   }
  NCPA   } Necessary Condition Procedures

  NECOUT} Output

```

4) Select from the available desirable condition modules (or write his own for additional criteria) and write the desirable condition portion of the program:

```

  DESINP} Input
  DLKLNRT}
  DESDYAD} Desirable Conditions

  DESOUT} Output

```

- 5) With the use of the program charts of Appendix C, coordinate input and output files between modules.

These modules are completely compatible with existing Pollock dyad-analysis routines and any others following a similar input/output format. For example, an optimization routine could be immediately followed by a dyad-analysis routine within the same program to perform a detailed dynamic analysis of the optimal linkage.

Table 7-1

Program Modules

<u>Synthesis</u>	<u>Necessary Conditions</u>	<u>Desirable Conditions</u>
SYNINP	NECINP	DESINP
SYNOUT	NECOUT	DESOUT
SCRVTRN	NCPA	DPVTICA
SINVERT	NCPB	DLKLNRT
SPTHCOG	NCPD	DLKSIZE
SANGCOG		
SDYDTYP		
	NCPG	DESDYAD
	NCPH	Overall Boundary,
	NCPJ	g-function
	NCPK	Vh-function
	NCPN	hrms-function
	NDYDTYP	VTorque
	NECDYAD	Shock Level
	Connection, Disconnection	
	g-fn. reversal, Encirclement, Non-encirclement	
<u>Support Routines</u>		
INITIAL		
ANALYZE		
ANLZPCG		
EXTRACT		

CHAPTER VIII RECOMMENDATIONS AND CONCLUSIONS

There are several logical extensions of the presented work. For the most part, they involve expansions of the underlying theories to reduce the levels of constraint in the choices of motion specification type and solution linkage type.

A. Recommendations

1.) Alternate Synthesis Procedures

As has been previously mentioned, it is desirable to keep the number of free parameters to 1, 2 or 3. With conventional 4MSP synthesis procedures (Chapter IV, Appendix A) this range is exceeded for problems #1 and #12. In problem #1 the number of free parameters is five, but is reduced to two by arbitrarily selected coupler angles in the motion specifications set. The number of free parameters in problem #12 is four, resulting in a four-dimensional grid and escalated computational complexity and cost.

All of the synthesis procedures presented in Chapter IV and Appendix A involve at least one application of the first tool of synthesis, the curvature transform, for 4MSP. Alternate synthesis procedures have been developed which allow

slightly different types of motion specifications to be made. For example, Sarkisyan, Gupta and Roth [124] have extended curvature transform theory to satisfy motion specifications in a least-squares minimum-error sense. Sutherland [125] has treated mixed exact-approximate specifications, again making use of least-squares approximations. Kramer and Sandor [48] have introduced "Selective Precision Synthesis (SPS)," a technique which produces dyadic constructions of mechanisms which satisfy "accuracy neighborhoods" about specified pivots.

Any synthesis procedure which 1.) produces linkage solution sets which can be mapped onto orthogonal grids and 2.) allows resynthesis within specified regions of the grid, can be used in the proposed optimization procedure. The availability of alternative synthesis procedures would enhance control of the number of optimization parameters.

2.) Alternate Solution Linkages

Another way to control the number of optimization parameters is to allow the use of different types of linkages. Synthesis procedures are presented for four-bar and six-bar linkages, which provide eight and fourteen free parameters respectively. Untreated here but also directly synthesizable are the slider-crank and inverted slider-crank (7 parameters), geared five-bar (12 parameters) and numerous variations of the six-bar using sliders and oscillating

blocks. In addition, four-bars have been designated as solution linkages for the first five problem classes, but more complex linkages could conceivably be used to provide more free parameters for these five classes.

3.) Alternate Modes of Cyclicity

It has been explained that the necessary condition procedure for a particular problem depends upon the desired type of motion in the solution linkage, and this dependency is the reason for the creation of the concept of mode of cyclicity. Restrictions are made in Chapter III which have the effect of limiting the modes of cyclicity under consideration to those for which there need be no supplementary knowledge of system dynamics in order to ensure control of motion through dead-center positions. These restrictions might be termed "geometric restrictions."

Many more modes of cyclicity are conceivable, some of which cannot exist for physical reasons, and some of which can exist but violate the geometric restrictions. Most of the physically realizable modes of cyclicity can be treated by the computational modules already presented. For example, a Grashof double-rocker with fully rotating coupler violates the geometric restrictions and is therefore not considered here as an acceptable mode of cyclicity, but a designer who could control this type of mechanism in a particular application could use Necessary Condition Procedure C (NCPC, Appendix C) to design one.

Other subsets of allowable modes of cyclicity can and should be defined and compiled, like those of Figures 3-4, 3-5 and 3-6 for the geometric restrictions. For example, there might be "dynamic" restrictions, "two-input" restrictions, and so on.

4.) Multiple Modes of Cyclicity

In putting together the computational modules of Appendix C for a particular problem, the user is constrained to select a single mode of cyclicity in order to define the appropriate necessary condition procedure. For many applications more than one of the available modes of cyclicity would be of interest.

It is sufficient to generate separate necessary condition grids for each of the selected modes of cyclicity, and combine these at the appropriate times (generally, before the necessary condition output, or the necessary condition dyad routine). The present modules have this capability (NECOUT and NECDYAD of Appendix C).

A better approach would consider the minimum range of conditions to be satisfied, and generate a single necessary condition procedure for the selected modes of cyclicity. This would be considerably more efficient due to the elimination of computational duplications at overlapping grid regions.

B. Conclusions

A unified approach to the design and optimization of four-bar and six-bar linkages has been presented. By isolating and defining the synthesis procedures and necessary condition procedures required (Appendix A,B), optimization relative to a broad range of design criteria can be performed.

A computational design based on a pool of modules of equivalent program hierarchy (Appendix C) has been developed and presented. The optimization routine for a particular problem is created through an appropriate coordination of these modules, as displayed by the examples of Appendix D.

Hopefully, the analytical and computational methodology introduced herein will provide the impetus for future efforts towards practical mechanism design and optimization.

APPENDIX A

PROBLEM TYPES AND SOLUTION LINKAGES

Use of Appendix A

Each part of Appendix A contains a drawing and brief description of a motion specification type, a drawing (with link and pivot notation) of solution linkage(s) and a small section describing each possible linkage solution. Each of these sections contains a moving-body/linkage-component match-up on the left and a corresponding synthesis procedure on the right (other match-ups are sometimes possible by switching conceptually identical bodies in the motion specification). Synthesis procedures are generally not unique.

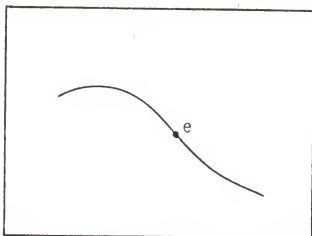
The solution linkages considered here are those which best provide an acceptable number of free parameters for optimization after a 4MSP synthesis. Geared five-bar and six-bar solutions to the simpler problems are not included: for more information on these types of solutions reference [105] is recommended.

Notation

- ". . . is attached to . . ."
- = ". . . is equivalent to . . ."

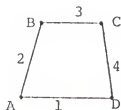
- * Necessary Condition Analysis covered in Appendix B.
- † Independent syntheses.
- "PATH COGNATES" Tools of synthesis are capitalized.
- "find 2" Motion of Link 2 is derivable for 4MSP, even though link shape may not yet be completely definable (if E is a ternary link).
- "INVERSION to 4" 4 is the new reference link.
- 7' The apostrophe indicates that link 7' is temporary and does not appear on the linkage drawing.
- [4MSP Grid:(2)x1-D] The grids produced by a 4MSP synthesis and necessary condition analysis will take the shape of . . . (here, (2) 1-dimensional grids). For path cognate problems, "L,R 2-D" indicates that 2-dimensional grids for both "left" and "right" cognates are generated.

Problem #1: PATH SYNTHESIS



The coplanar motion
of point e is con-
trolled.

SOLUTION LINKAGE



FOUR-BAR

Moving body/Linkage Component Match

Four-Bar

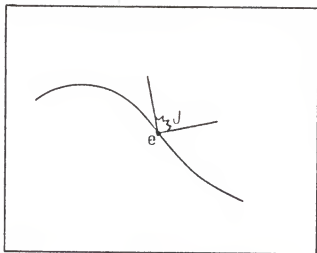
Link 1 is grounded
Point e → Link 3.

Synthesis Procedure

*† CURVATURE TRANSFORM to
find 4,2.

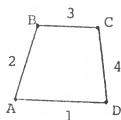
[4MSP Grid: 2-D]

Problem #2: MOTION SYNTHESIS



The coplanar motion of plane J is controlled.

SOLUTION LINKAGE



FOUR BAR

Four-Bar

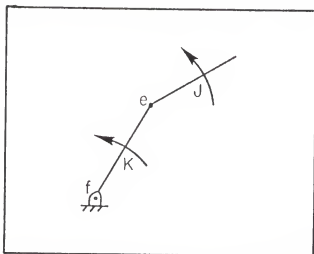
Link 1 is grounded.
Plane J $\rightarrow$ Link 3.

Synthesis Procedure

*† CURVATURE TRANSFORM to find 4,2.

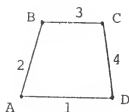
[4MSP Grid: 2-D]

Problem #3: ANGULAR COORDINATION OF A TWO LINK CHAIN



The rotation of crank K is coordinated with the coplanar motion of attached link J.

SOLUTION LINKAGE



FOUR-BAR

Four-Bar

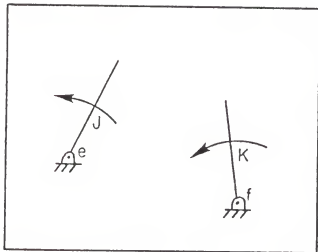
Link 1 is grounded.
Plane J $\rightarrow$ Link 3 and
Point e = Pivot B.
Crank K = Link 2 and
Point f = Pivot A.

Synthesis Procedure

*† CURVATURE TRANSFORM to
find 4.

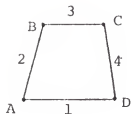
[4MSP Grid: 1-D]

Problem #4: ANGULAR COORDINATION OF TWO CRANKS



The rotation of crank J is coordinated with the rotation of crank K.

SOLUTION LINKAGE



FOUR-BAR

Four-Bar

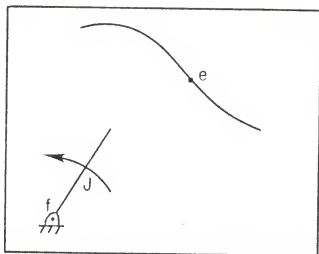
Link 1 is grounded.
 Crank J $\rightarrow$ Link 2 and
 Point e = Pivot A.
 Crank K $\rightarrow$ Link 4 and
 Point f = Pivot D.

Synthesis Procedure

*† INVERSION to 2.
 CURVATURE TRANSFORM TO
 find 1,3.

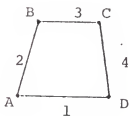
[4MSP Grid: 1-D]

Problem #5: PATH COORDINATION WITH A CRANK



The coplanar motion of point e is coordinated with the rotation of crank J.

SOLUTION LINKAGE



FOUR-BAR

Four-Bar

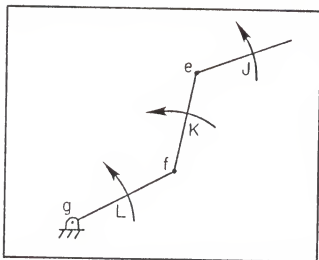
Link 1 is grounded
Point e $\rightarrow$ Link 3.
Crank J = Link 2.
Point f = Pivot A.

Synthesis Procedure

*† CURVATURE TRANSFORM and
PATH COGNATES to find
2, 4.

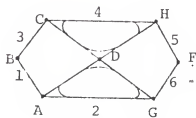
[4MSP Grid: L,R 2-D]

Problem #6: ANGULAR COORDINATION OF A THREE-LINK CHAIN

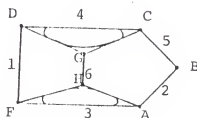


The coplanar motion of planes J and K and crank L are coordinated.

SOLUTION LINKAGES



WATT



STEPHENSON

Synthesis Procedures

Watt 1 - Link 1 is grounded.

- Plane J $\rightarrow$ Link 5 and Point e = Pivot F.
Plane K = Link 6 and Point f = Pivot G.
Crank L = Link 2 and Point g = Pivot A.
- Plane J $\rightarrow$ Link 5 and Point e = Pivot H.
Plane K = Link 4 and Point f = Pivot C.
Crank L = Link 3 and Point g = Pivot B.

INVERSION to 2.
CURVATURE TRANSFORM to find 4.
INVERSION to 1.
CURVATURE TRANSFORM to find 3.

[4MSP Grid:(2)x1-D]

CURVATURE TRANSFORM to find 2.
INVERSION to 2.
CURVATURE TRANSFORM to find 6.

[4MSP Grid:(2)x1-D]

- c.) Plane J $\rightarrow$ Link 5 and
 Point e = Pivot H.
 Plane K = Link 4 and
 Point f = Pivot D.
 Crank L = Link 2 and
 Point g = Pivot A.

*† CURVATURE TRANSFORM to
 find 3.
 INVERSION to 2.
 CURVATURE TRANSFORM to
 find 6.

[4MSP Grid:(2)x1-D]

Stephenson 1 - Link 1 is grounded.

- Plane J $\rightarrow$ Link 5 and
 Point e = Pivot B.
 Plane K = Link 2 and
 Point f = Pivot A.
 Crank L = Link 3 and
 Point g = Pivot F.

CURVATURE TRANSFORM to
 find 4.
 INVERSION to 3.
 CURVATURE TRANSFORM to
 find 6.

[4MSP Grid:(2)x1-D]

Stephenson 2 - Link 2 is grounded.

- a.) Plane J $\rightarrow$ Link 4 and
 Point e = Pivot D.
 Plane K = Link 1 and
 Point f = Pivot F.
 Crank L = Link 3 and
 Point g = Pivot A.

† CURVATURE TRANSFORM to
 find 5.
 INVERSION to 3.
 CURVATURE TRANSFORM to
 find 6.

[4MSP Grid:(2)x1-D]

- b.) Plane J $\rightarrow$ Link 1 and
 Point e = Pivot D.
 Plane K = Link 4 and
 Point f = Pivot C.
 Crank L = Link 5 and
 Point g = Pivot B.

CURVATURE TRANSFORM to
 find 3.
 INVERSION to 4.
 CURVATURE TRANSFORM to
 find 6.

[4MSP Grid:(2)x1-D]

Stephenson 3 - Link 3 is grounded.

- a.) Plane J $\rightarrow$ Link 5 and
 Point e = Pivot C.
 Plane K = Link 4 and
 Point f = Pivot D.
 Crank L = Link 1 and
 Point g = Pivot F.

† CURVATURE TRANSFORM to
 find 2.
 CURVATURE TRANSFORM to
 find 6.

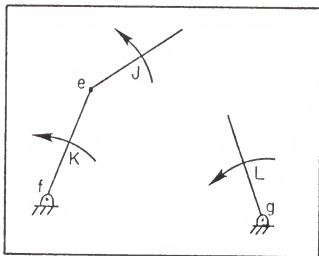
[4MSP Grid:(2)x1-D]

- b.) Plane J $\rightarrow$ Link 4 and
 Point e = Pivot C.
 Plane K = Link 5 and
 Point f = Pivot 3.
 Crank L = Link 2 and
 Point g = Pivot A.

† CURVATURE TRANSFORM to
 find 1,6.

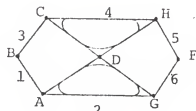
[4MSP Grid: 2-D]

Problem #7: ANGULAR COORDINATION OF A CRANK AND A TWO-LINK CHAIN

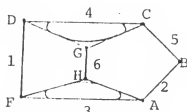


The coplanar motion of two-link chain JK is coordinated with the rotation of crank L.

SOLUTION LINKAGES



WATT



STEPHENSON

Watt 1 - Link 1 is grounded.

Plane J $\rightarrow$ Link 6 and
Point e = Pivot A.
Crank K = Link 2 and
Point f = Pivot A.
Crank L $\rightarrow$ Link 3 and
Point g = Pivot B.

Synthesis Procedures

INVERSION to 2.
CURVATURE TRANSFORM to find 4.
INVERSION to 6.
CURVATURE TRANSFORM to find 5.

[4MSP Grid:(2)x1-D]

Watt 2 - Link 2 is grounded.

Plane J → Link 3 and	CURVATURE TRANSFORM to find 4.
Point e = Pivot B.	INVERT to 6.
Crank K = Link 1 and	CURVATURE TRANSFORM to find 5.
Point f = Pivot A.	
Crank L → Link 6 and	[4MSP Grid: (2)x1-D]
Point g = Pivot G.	

Stephenson 1 - Link 1 is grounded.

Plane J → Link 2 and	*† INVERSION to 4.
Point e = Pivot A.	CURVATURE TRANSFORM to find 6.
Crank K = Link 3 and	CURVATURE TRANSFORM to find 5.
Point f = Pivot F.	
Crank L → Link 4 and	[4MSP Grid: (2)x1-D]
Point g = Pivot D.	

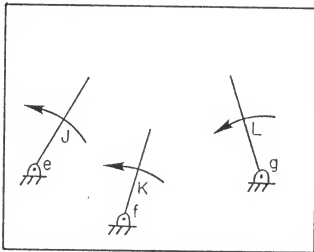
Stephenson 2 - Link 2 is grounded.

- | | |
|--------------------------|--------------------------------|
| a.) Plane J → Link 1 and | INVERSION to 1. |
| Point e = Pivot F. | CURVATURE TRANSFORM to find 4. |
| Crank K = Link 3 and | INVERSION to 3. |
| Point f = Pivot A. | CURVATURE TRANSFORM to find 6. |
| Crank L → Link 5 and | |
| Point g = Pivot B. | [4MSP Grid: (2)x1-D] |
| b.) Plane J → Link 4 and | † INVERSION to 3. |
| Point e = Pivot G. | CURVATURE TRANSFORM to find |
| Crank K = Link 5 and | 1, 6. |
| Point f = Pivot B. | |
| Crank L → Link 3 and | [4MSP Grid: 2-D] |
| Point g = Pivot A. | |

Stephenson 3 - Link 3 is grounded.

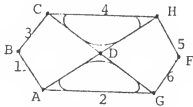
- | | |
|--------------------------|----------------------------------|
| a.) Plane J → Link 4 and | † CURVATURE TRANSFORM to find 6. |
| Point e = Pivot D. | INVERSION to 2. |
| Crank K = Link 1 and | CURVATURE TRANSFORM to find 5. |
| Point f = Pivot F. | |
| Crank L → Link 2 and | [4MSP Grid: (2)x1-D] |
| Point g = Pivot A. | |
| b.) Plane J → Link 5 and | INVERSION to 1. |
| Point e = Pivot B. | CURVATURE TRANSFORM to find 4. |
| Crank K = Link 2 and | INVERSION to 3. |
| Point f = Pivot A. | CURVATURE TRANSFORM to find 6. |
| Crank L → Link 1 and | |
| Point g = Pivot F. | [4MSP Grid: (2)x1-D] |

Problem #8: ANGULAR COORDINATION OF THREE CRANKS

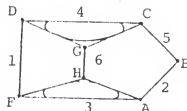


The rotations of cranks J, K and L are coordinated.

SOLUTION LINKAGES



WATT



STEPHENSON

Watt 2 - Link 2 is grounded.

Crank J → Link 1 and
Point e = Pivot A.
Crank K → Link 4 and
Point f = Pivot D.
Crank L → Link 6 and
Point g = Pivot G.

*† INVERSION to 4.
CURVATURE TRANSFORM to find 3.
CURVATURE TRANSFORM to find 5.

[4MSP Grid: (2)x1-D]

Synthesis Procedures

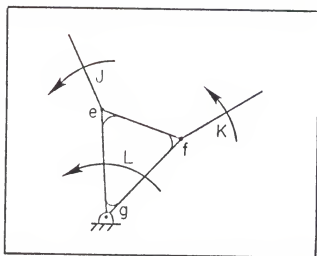
Stephenson 3 - Link 3 is grounded.

Crank J → Link 1 and
Point e = Pivot F.
Crank K → Link 6 and
Point f = Pivot H.
Crank L → Link 2 and
Point g = Pivot A.

INVERSION to 1.
CURVATURE TRANSFORM to find 4.
INVERSION to 2.
CURVATURE TRANSFORM to find 5.

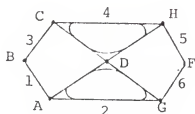
[4MSP Grid:(2)x1-D]

Problem #9: ANGULAR COORDINATION OF THREE ADJACENT PLANES

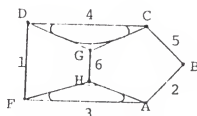


The coplanar motions of planes J and K are coordinated with the rotation of L.

SOLUTION LINKAGES



WATT



STEPHENSON

Watt 1 - Link 1 is grounded.

Plane J $\rightarrow$ Link 4 and
Point e = Pivot D.
Plane K $\rightarrow$ Link 6 and
Point f = Pivot G.
Plane L = Link 2 and
Point g = Pivot A.

Synthesis Procedures

† CURVATURE TRANSFORM to find 3.
INVERSION to 6.
CURVATURE TRANSFORM to find 5.

[4MSP Grid: (2)x1-D]

Watt 2 - Link 2 is grounded.

Plane J → Link 3 and	†* CURVATURE TRANSFORM to find 1.
Point e = Pivot C.	CURVATURE TRANSFORM to find 6.
Plane K → Link 5 and	
Point f = Pivot H.	[4MSP Grid:(2)x1-D]
Plane L = Link 4 and	
Point g = Pivot D.	

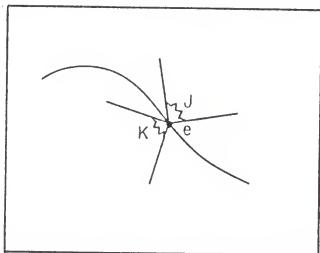
Stephenson 1 - Link 1 is grounded.

Plane J → Link 6 and	CURVATURE TRANSFORM to find 4.
Point e = Pivot H.	INVERSION to 2.
Plane K → Link 2 and	CURVATURE TRANSFORM to find 5.
Point f = Pivot A.	
Plane L = Link 3 and	[4MSP Grid:(2)x1-D]
Point g = Pivot F.	

Stephenson 2 - Link 2 is grounded.

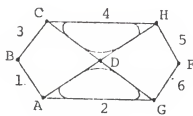
Plane J → Link 1 and	INVERSION to 1.
Point e = Pivot F.	CURVATURE TRANSFORM to get 4.
Plane K → Link 6 and	INVERSION to 2.
Point f = Pivot H.	CURVATURE TRANSFORM to get 5.
Plane L = Link 3 and	
Point g = Pivot A.	[4MSP Grid:(2)x1-D]

Problem #10: DOUBLE COPLANAR SYNTHESIS OF TWO
ADJACENT PLANES

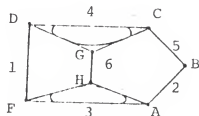


The coplanar motions of Plane J and Plane K (connected at pivot e) are coordinated.

SOLUTION LINKAGES



WATT



STEPHENSON

Watt 1 - Link 1 is grounded.

- a.) Plane J $\rightarrow$ Link 4.
Plane K $\rightarrow$ Link 5.
Point e = Pivot H.

- b.) Plane J $\rightarrow$ Link 5.
Plane K $\rightarrow$ Link 6.
Point e = Pivot F.

Synthesis Procedures

CURVATURE TRANSFORM to find 2,3.

INVERSION to 2.

CURVATURE TRANSFORM to find 6.

[4MSP Grid: 2-D+1-D]

CURVATURE TRANSFORM to find 2.

INVERSION to 2.

CURVATURE TRANSFORM to find 4.

INVERSION to 1.

CURVATURE TRANSFORM to find 3.

[4MSP Grid: (3)x1-D]

Stephenson 1 - Link 1 is grounded.

Plane J → Link 5.	CURVATURE TRANSFORM to find 3.
Plane K → Link 2.	CURVATURE TRANSFORM to find 4.
Point e = Pivot B.	INVERSION to 3.
	CURVATURE TRANSFORM to find 6.

[4MSP Grid: (3)x1-D]

Stephenson 2 - Link 2 is grounded.

Plane J → Link 1.	CURVATURE TRANSFORM to find 3.
Plane K → Link 4.	CURVATURE TRANSFORM to find 5.
Point e = Pivot D.	INVERSION to 3.
	CURVATURE TRANSFORM to find 6.

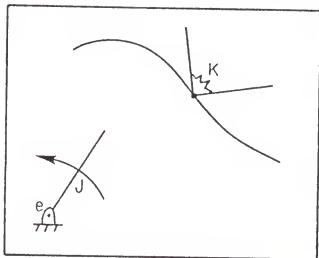
[4MSP Grid: (3)x1-D]

Stephenson 3 - Link 3 is grounded.

Plane J → Link 4.	*† CURVATURE TRANSFORM to find
Plane K → Link 5.	1,6
Point e = Pivot C.	CURVATURE TRANSFORM to find 2.

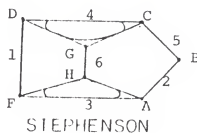
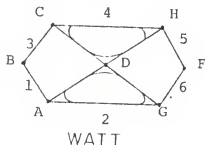
[4MSP Grid: 2-D+1-D]

Problem #11: COPLANAR MOTION COORDINATED WITH A CRANK



The coplanar motion of Plane K is coordinated with the rotation of crank J.

SOLUTION LINKAGES



Watt 1 - Link 1 is grounded

- a.) Crank J → Link 2 and
Point e = Pivot A.
Plane K → Link 5.

- b.) Crank J → Link 3 and
Point e = Pivot B.
Plane K → Link 5.

Synthesis Procedures

INVERSION to 2.
CURVATURE TRANSFORM to find
4, 6.
INVERSION to 1.
CURVATURE TRANSFORM to find 3.

[4MSP Grid: 2-D+1-D]

INVERSION to 3.
CURVATURE TRANSFORM to find 4.
INVERSION to 1.
CURVATURE TRANSFORM to find 2.
INVERSION to 2.
CURVATURE TRANSFORM to find 6.

[4MSP Grid: (3)x1-D]

c.) Crank J → Link 3.
 Point e = Pivot B.
 Plane K → Link 6.

CURVATURE TRANSFORM to find 2.
 INVERSION to 2.
 CURVATURE TRANSFORM to find 4.
 INVERSION to 4.
 CURVATURE TRANSFORM to find 5.

[4MSP Grid: (3)x1-D]

Watt 2 - Link 2 is grounded.

Crank J → Link 1.
 Point e = Pivot A.
 Plane K → Link 5.

CURVATURE TRANSFORM to find
 4,6.
 INVERSION to 1.
 CURVATURE TRANSFORM to find 3.

[4MSP Grid: 2-D+1-D]

Stephenson 1 - Link 1 is grounded.

Crank J → Link 3.
 Point e = Pivot F.
 Plane K → Link 5.

CURVATURE TRANSFORM to find 4.
 INVERSION to 3
 CURVATURE TRANSFORM to find 2.
 CURVATURE TRANSFORM to find 6.

[4MSP Grid: (3)x1-D]

Stephenson 2 - Link F is grounded.

a.) Crank J → Link 3.
 Point e = Pivot A.
 Plane K → Link 4.

*† CURVATURE TRANSFORM to find 5.
 INVERSION to 3.
 CURVATURE TRANSFORM to find
 1,6.

[4MSP Grid: 1-D+2-D]

b.) Crank J → Link 5.
 Point e = Pivot B.
 Plane K → Link 1.

CURVATURE TRANSFORM to find 3.
 INVERSION to 5.
 CURVATURE TRANSFORM to find 4.
 INVERSION to 4.
 CURVATURE TRANSFORM to find B.

[4MSP Grid: (3)x1-D]

Stephenson 3 - Link 3 is grounded.

a.) Crank J → Link 1.
 Point e = Pivot F.
 Plane K → Link 5.

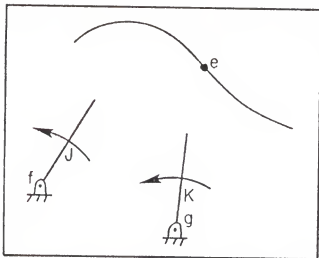
CURVATURE TRANSFORM to find 2.
 INVERSION to 1.
 CURVATURE TRANSFORM to find 4.
 INVERSION to 3.
 CURVATURE TRANSFORM to find 6.

[4MSP Grid: (3)x1-D]

b.) Crank J → Link 2.
Point e = Pivot A.
Plane K → Link 4.

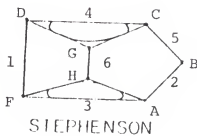
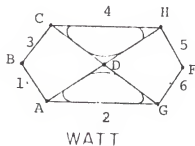
† CURVATURE TRANSFORM to find
1,6.
INVERSION to 2.
CURVATURE TRANSFORM to find 5.
[4MSP Grid:2-D+1-D]

Problem #12: POINT PATH COORDINATION WITH TWO CRANKS



The coplanar motion of point e and the rotations of cranks J and K are coordinated.

SOLUTION LINKAGES



Watt 1 - Link 1 is grounded.

- a.) Crank J $\rightarrow$ Link 3 and
Point f = Pivot B.
Crank K $\rightarrow$ Link 2.
Point e on Link 3.

Synthesis Procedures

CURVATURE TRANSFORM to find 4,6.

INVERSION to 1.

CURVATURE TRANSFORM and PATH COGNATES to find 3.

[4MSP Grid:1,R 2-D, and 1-D]

- b.) Crank J → Link 3 and
 Point f = Pivot B.
 Crank K → Link 2.
 Point e on Link 6.

CURVATURE TRANSFORM and PATH
 COGNATES to find 2,6, dummy 7'.
 INVERSION to 2.
 CURVATURE TRANSFORM to find 4.
 INVERSION to 4.
 CURVATURE TRANSFORM to find 5.

[4MSP Grid:L,R 2-D, +(2)x1-D]

Watt 2 - Link 2 is grounded.

- Crank J → Link 1 and
 Point f = Pivot A.
 Crank K → Link 4.
 Point e on Link 5.

* CURVATURE TRANSFORM and PATH
 COGNATES to find 6,4.
 INVERSION to 1.
 CURVATURE TRANSFORM to find 3.

[4MSP Grid:L,R2-D,+1-D]

Stephenson 1 - Link 1 is grounded.

- Crank J → Link 4 and
 Crank K → Link 3.
 Point g = Pivot F.
 Point e on Link 5.

CURVATURE TRANSFORM and PATH
 COGNATES to find 4, dummy 7'.
 INVERSION to 3.
 CURVATURE TRANSFORM to find 6.
 CURVATURE TRANSFORM to find 2.

[4MSP Grid:L,R 2-D, +(2)x1-D]

Stephenson 2 - Link 2 is grounded.

- a.) Crank J → Link 3 and
 Point f = Pivot A.
 Crank K → Link 5.
 Point e on Link 4.

CURVATURE TRANSFORM and PATH
 COGNATES to find H, dummy 7'.
 INVERSION to 3.
 CURVATURE TRANSFORM to find
 1,6.

[4MSP Grid:L,R 2-D, +2-D]

- b.) Crank J → Link 3.
 Crank K → Link 5 and
 Point g = Pivot B.
 Point e on Link 1.

CURVATURE TRANSFORM and PATH
 COGNATES to find 3, dummy 7'.
 INVERSION to 5.
 CURVATURE TRANSFORM to find 4.
 INVERSION to 3.
 CURVATURE TRANSFORM to find 6.

[4MSP Grid:L,R 2-D, +(2)x1-D]

Stephenson 3 - Link 3 is grounded.

- a.) Crank J → Link 1 and
 Point f = Pivot F.
 Crank K → Link 2.
 Point e on Link 5.

CURVATURE TRANSFORM and PATH
 COGNATES to find 2, dummy 7'.
 INVERSION to 1.
 CURVATURE TRANSFORM to find 4.
 INVERSION to 3.
 CURVATURE TRANSFORM to find 6.

[4MSP GRID: L,R 2-D, +(2)x1-D]

- b.) Crank J → Link 1.
 Crank K → Link 2 and
 Point g = Pivot A.
 Point e on Link 4.

CURVATURE TRANSFORM and PATH
 COGNATES to find 1,6.
 INVERSION to 2.
 CURVATURE TRANSFORM to find 5.

[4MSP Grid: L,R 2-D, +1-D]

APPENDIX B

MODES OF CYCLICITY AND
NECESSARY CONDITION PROCEDURES

Use of Appendix B

Each part of Appendix B contains a problem type, a linkage solution and a corresponding synthesis procedure. The solutions have been selected so as to show at least one example of a solution for every type of solution linkage.

The contents of Appendix B are designed to provide a guide for the writing of the necessary condition part of an optimization program, for all twelve problem types. For an implicit understanding of the procedures, the reader is referred to Chapter 5.

Notation



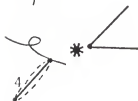
Driven link.



The links attached to the circled pivot may have complete relative rotation.



The links attached to the pivot cannot have complete relative rotation.



The pivot may not pass through a change point.

Link 4 is synthesized and selected from a circle point cubic curve.

"CURVATURE TRANSFORM"

Tools of synthesis are capitalized.

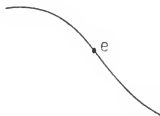
"find 2"

Motion of link B is derivable for 4MSP, even though (if B is ternary) link shape may not yet be completely definable.

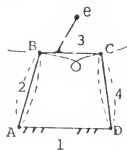
"INVERSION to 3"

3 is the new reference link.

Problem #1
Path Synthesis



Linkage Solution Synthesis Procedure

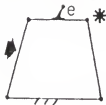


FOUR-BAR

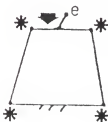
- 1) CURVATURE
TRANSFORM to
find 2,4.

Modes of Cyclicity

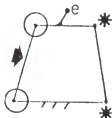
1)



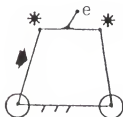
2)



3)



4)



Necessary Condition Procedures

- a) Necessary Condition Procedure D for 1234.

- a) Necessary Condition Procedure H for 1234.

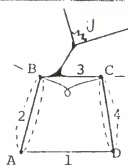
- a) Necessary Condition Procedure A for 1234.

- a) Necessary Condition Procedure B for 1234.

Problem #2
Motion Synthesis



Linkage Solution



FOUR-BAR

Synthesis Procedure

- 1) CURVATURE
TRANSFORM
to find 2,4.

Modes of Cyclicity

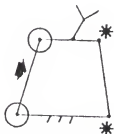
1)



2)



3)



4)



Necessary Condition Procedures

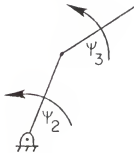
- a) Necessary Condition Procedure D for 1234.

- a) Necessary Condition Procedure H for 1234.

- a) Necessary Condition Procedure A for 1234.

- a) Necessary Condition Procedure B for 1234.

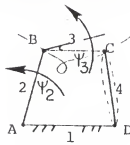
Problem #3
Angular Coordi-
nation of a Two-
Link Chain



Linkage Solution

Synthesis Procedure

- 1) CURVATURE
 TRANSFORM
 to find 4.



FOUR-BAR

Pivots , pre-
 specified.

Modes of Cyclicity

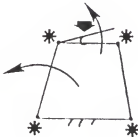
Necessary Condition Procedures

1)



- a) Necessary Condition Pro-
 cedure D for 1234.

2)



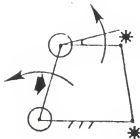
- a) Necessary Condition Pro-
 cedure H for 1234.

3)



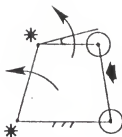
- a) Necessary Condition Pro-
 cedure D for 1234.

4)



- a) Necessary Condition Pro-
 cedure A for 1234.

5)



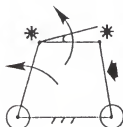
a) Necessary Condition Procedure A for 1234.

6)



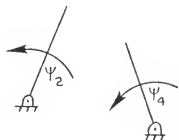
a) Necessary Condition Procedure B for 1234.

7)

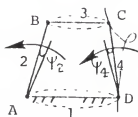


a) Necessary Condition Procedure B for 1234.

Problem #4
Angular Coordination
of Two Cranks



Linkage Solution Synthesis Procedure



FOUR-BAR

Pivots A, D
prespecified.

1) INVERSION to 2.

2) CURVATURE
TRANSFORM
to find 1, 3.

Modes of Cyclicity

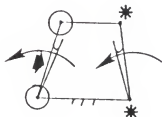
1)



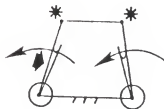
2)



3)



4)



Necessary Condition Procedures

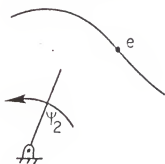
a) Necessary Condition Procedure D for 1234.

a) Necessary Condition Procedure H for 1234.

a) Necessary Condition Procedure B for 1234.

a) Necessary Condition Procedure A for 1234.

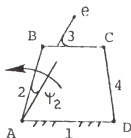
Problem #5
Path Coordination
with a Crank



Linkage Solution

Synthesis Procedure

- 1) CURVATURE
 TRANSFORM and
 PATH COGNATES
 to find 2,4.

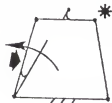


FOUR-BAR

Modes of Cyclicity

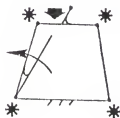
Necessary Condition Procedures

1)



- a) Necessary Condition Procedure M for 1234.

2)



- a) Necessary Condition Procedure N for 1234.

3)



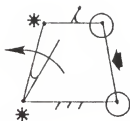
- a) Necessary Condition Procedure L for 1234.

4)



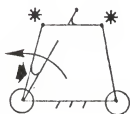
- a) Necessary Condition Procedure I for 1234.

5)



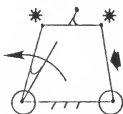
a) Necessary Condition Procedure J for 1234.

6)



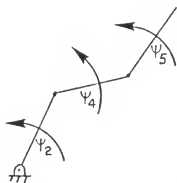
a) Necessary Condition Procedure K for 1234.

7)



a) Necessary Condition Procedure K for 1234.

Problem #6
Angular Coordination
of a Three-Link
Chain.



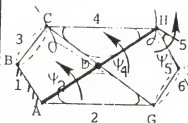
Linkage Solution

Synthesis Procedure

1) CURVATURE TRANS-
FORM to find 3.

2) INVERSION to 2.

6) 3) CURVATURE TRANS-
FORM to find 6.

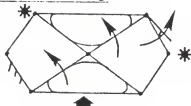


WATT 1

Pivots A, D, H
prespecified.

Modes of Cyclicity

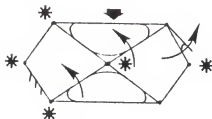
1)



2)



3)



4)



Necessary Condition Procedures

a) Necessary Condition Pro-
cedure D for 1234.

b) Necessary Condition Pro-
cedure D for 2456.

a) Necessary Condition Pro-
cedure D for 1234.

b) Necessary Condition Pro-
cedure D for 2456.

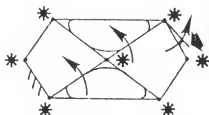
a) Necessary Condition Pro-
cedure H for 1234.

b) Necessary Condition Pro-
cedure D for 2456.

a) Necessary Condition Pro-
cedure H for 1234.

b) Necessary Condition Pro-
cedure H for 2456.

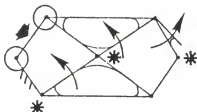
5)



a) Necessary Condition Procedure II for 1234.

b) Necessary Condition Procedure II for 2456.

6)

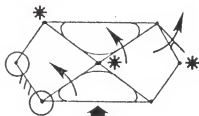


a) Necessary Condition Procedure A for 1234.

b) Necessary Condition Procedure D for 2456.

c) Verify connection of 56.

7)

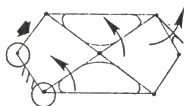


a) Necessary Condition Procedure B for 1234.

b) Necessary Condition Procedure D for 2456.

c) Verify connection of 56.

8)

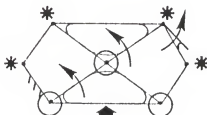


a) Necessary Condition Procedure B for 1234.

b) Necessary Condition Procedure D for 2456.

c) Verify connection of 56.

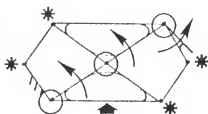
9)



a) Necessary Condition Procedure A for 1234.

b) Necessary Condition Procedure B for 2456.

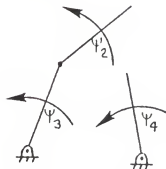
10)



a) Necessary Condition Procedure A for 1234.

b) Necessary Condition Procedure A for 2456.

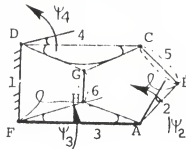
Problem #7
Angular Coordina-
tion of a Crank
and a Two-Link
Chain.



Linkage Solution

Synthesis Procedure

- 1) INVERSION to 4.
- 2) CURVATURE TRANS-
FORM to find 6.
- 3) CURVATURE TRANS-
FORM to find 2.

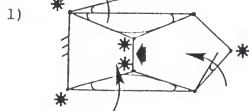


STEPHENSON 1

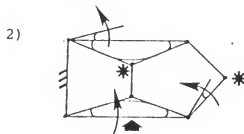
Pivots F, A, D
prespecified.

Modes of Cyclicity

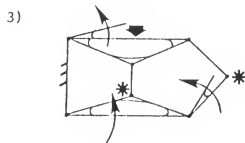
Necessary Condition Procedures



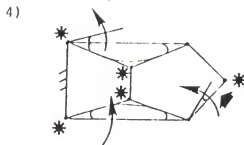
- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure H for 1346
- c) Verify Connection of 25.



- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure G for 1346.
- c) Verify Connection of 25.

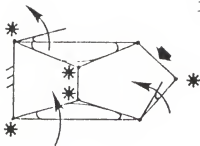


- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure D for 1346.
- c) Verify Connection of 25.



- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure H for 1346.
- c) Verify Connection of 25.

5)

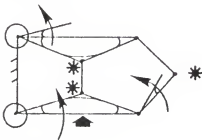


a) Necessary Condition Dyad Type (25).

b) Necessary Condition Procedure II for 1346.

c) Verify Connection of 25.

6)



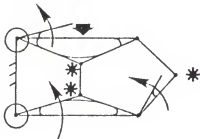
a) Necessary Condition Dyad Type (25).

b) Necessary Condition Procedure A for 1346.

c) Verify Connection of 25.

d) Verify Non-encirclement (AC).

7)



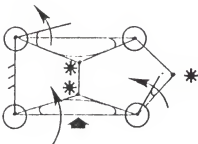
a) Necessary Condition Dyad Type (25).

b) Necessary Condition Procedure A for 1346.

c) Verify Connection of 25.

d) Verify Non-encirclement (AC).

8)



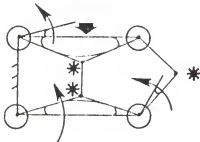
a) Necessary Condition Dyad Type (25).

b) Necessary Condition Procedure A for 1346.

c) Verify Connection of 25.

d) Verify Encirclement (AC).

9)



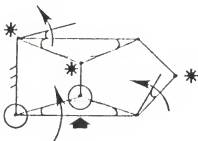
a) Necessary Condition Dyad Type (25).

b) Necessary Condition Procedure A for 1346.

c) Verify Connection of 25.

d) Verify Encirclement (AC).

10)



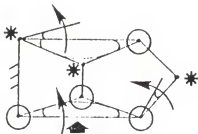
a) Necessary Condition Dyad Type (25).

b) Necessary Condition Procedure C for 1346.

c) Verify Connection of 25.

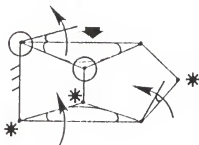
d) Verify Non-encirclement (AC).

11)



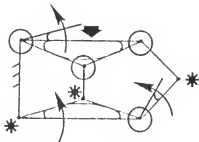
- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure C for 1346.
- c) Verify Connection of 25.
- d) Verify Encirclement (AC).

12)



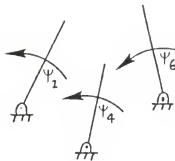
- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure B for 1346.
- c) Verify Connection of 25.
- d) Verify Non-encirclement (AC).

13)

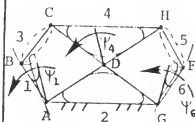


- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure B for 1346.
- c) Verify Connection of 25.
- d) Verify Encirclement (AC).

Problem #8
Angular Coordination
of Three
Crankes



Solution Linkage



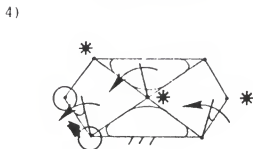
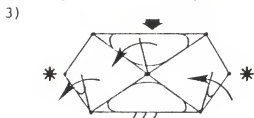
WATT 2

Pivots A,G,D
prespecified.

Synthesis Procedure

- 1) INVERSION to 4.
- 2) CURVATURE TRANSFORM to find 3.
- 3) CURVATURE TRANSFORM to find 5.

Modes of Cyclicity



Necessary Condition Procedures

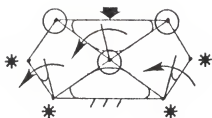
- a) Necessary Condition Procedure G for 1234.
- b) Necessary Condition Procedure D for 2456.

- a) Necessary Condition Procedure H for 1234.
- b) Necessary Condition Procedure D for 2456.

- a) Necessary Condition Procedure D for 1234.
- b) Necessary Condition Procedure D for 2456.

- a) Necessary Condition Procedure C for 1234.
- b) Necessary Condition Procedure D for 2456.
- c) Verify Connection of 56.

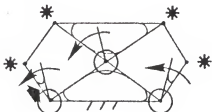
5)



a) Necessary Condition Procedure B for 1234.

b) Necessary Condition Procedure B for 2456.

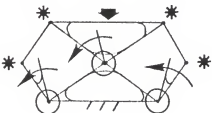
6)



a) Necessary Condition Procedure A for 1234.

b) Necessary Condition Procedure A for 2456.

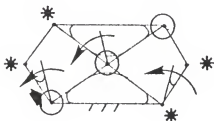
7)



a) Necessary Condition Procedure A for 1234.

b) Necessary Condition Procedure A for 2456.

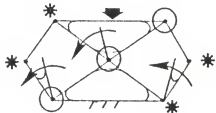
8)



a) Necessary Condition Procedure A for 1234.

b) Necessary Condition Procedure B for 2456.

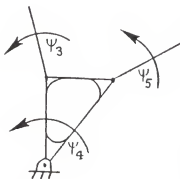
9)



a) Necessary Condition Procedure A for 1234.

b) Necessary Condition Procedure B for 2456.

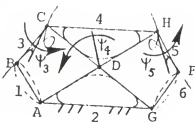
Problem #9
Angular Coordina-
tion of Three Ad-
acent Planes



Linkage Solution

Synthesis Procedure

- 1) CURVATURE TRANSFORM to find 1.
- 2) CURVATURE TRANSFORM to find 6.



WATT 2

Pivots H,C,D
 prespecified

Modes of Cyclicity

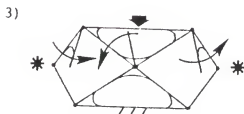
Necessary Condition Procedures



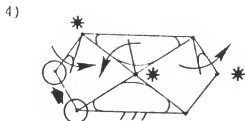
- a) Necessary Condition Procedure D for 1234.
- b) Necessary Condition Procedure D for 2456.



- a) Necessary Condition Procedure II for 1234.
- b) Necessary Condition Procedure D for 2456.

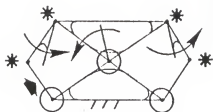


- a) Necessary Condition Procedure D for 1234.
- b) Necessary Condition Procedure D for 2456.



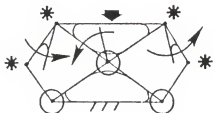
- a) Necessary Condition Procedure A for 1234.
- b) Necessary Condition Procedure D for 2456.
- c) Verify Connection of 56.

5)



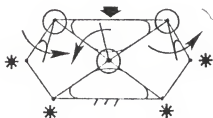
- a) Necessary Condition Procedure B for 1234.
- b) Necessary Condition Procedure B for 2456.

6)



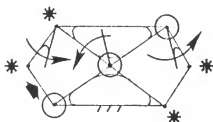
- a) Necessary Condition Procedure B for 1234.
- b) Necessary Condition Procedure B for 2456.

7)



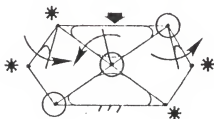
- a) Necessary Condition Procedure A for 1234.
- b) Necessary Condition Procedure A for 2456.

8)



- a) Necessary Condition Procedure B for 1234.
- b) Necessary Condition Procedure A for 2456.

9)



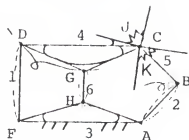
- a) Necessary Condition Procedure B for 1234.
- b) Necessary Condition Procedure A for 2456.

Problem #10
Double Coplanar
Synthesis of Two
Adjacent Planes



Linkage Solution

Synthesis Procedure



- 1) CURVATURE TRANSFORM to find 1,6.
- 2) CURVATURE TRANSFORM to find 2.

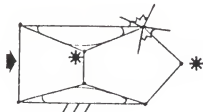
STEPHENSON 3

Pivot C
 prespecified.

Modes of Cyclicity

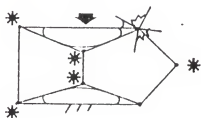
Necessary Condition Procedures

1)



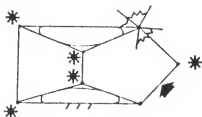
- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure D for 1346.
- c) Verify Connection of 25.

2)



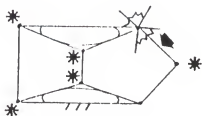
- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure H for 1346.
- c) Verify Connection of 25.

3)



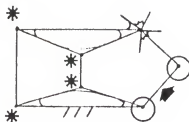
- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure H for 1346.
- c) Verify Connection of 25.
- d) Verify Non-q-function Reversal (2 relative to 3).

4)



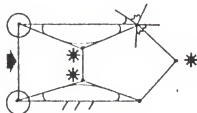
- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure H for 1346.
- c) Verify Connection of 25.

5)



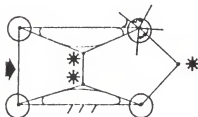
- a) Necessary Condition Procedure H for 1346.
- b) Verify Disconnection of 25. (2 short).

6)



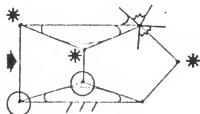
- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure A for 1346.
- c) Verify Connection of 25.
- d) Verify Non-encirclement (AC).

7)



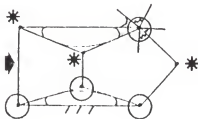
- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure A for 1346.
- c) Verify Connection of 25.
- d) Verify Encirclement (AC).

8)



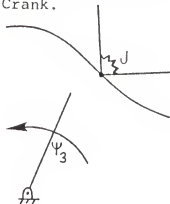
- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure B for 1346.
- c) Verify Connection of 25.
- d) Verify Non-encirclement (AC).

9)

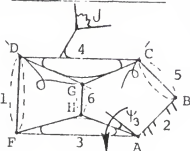


- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure B for 1346.
- c) Verify Connection of 25.
- d) Verify Encirclement (AC).

Problem #11
Coplanar Motion Co-
ordinated with a
Crank.



Linkage Solution



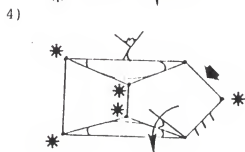
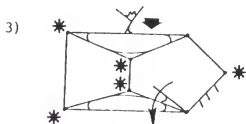
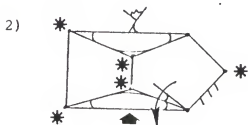
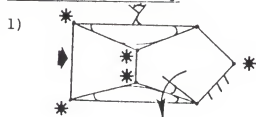
Synthesis Procedure

- 1) CURVATURE TRANSFORM to find 5.
- 2) INVERSION to 3.
- 3) CURVATURE TRANSFORM to find 1,6.

STEPHENSON 2

Pivot A
is prespecified.

Modes of Cyclicity



Necessary Condition Procedures

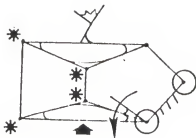
- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure II for 1346.
- c) Verify Connection of 25.

- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure II for 1346.
- c) Verify Connection of 25.
- d) Verify Non-g-function Reversal (2 relative to 3).

- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure II for 1346.
- c) Verify Connection of 25.

- a) Necessary Condition Dyad Type (25).
- b) Necessary Condition Procedure II for 1346.
- c) Verify Connection of 25.
- d) Verify Non-g-function Reversal (5 relative to 2).

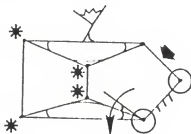
5)



a) Necessary Condition Procedure H for 1346.

b) Verify Disconnection of 25 (2-short).

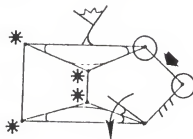
6)



a) Necessary Condition Procedure H for 1346.

b) Verify Disconnection of 25 (2-short).

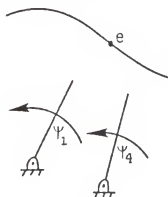
7)



a) Necessary Condition Procedure H for 1346.

b) Verify Disconnection of 25 (5-short).

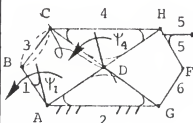
Problem #12
Path Coordination
with Two Cranks



Linkage Solution

Synthesis Procedure

- 1) CURVATURE TRANSFORM to find 4,6.
- 2) INVERSION to 1.
- 3) CURVATURE TRANSFORM to find 3.



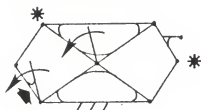
WATT 2

Pivot A
 prespecified.

Modes of Cyclicity

Necessary Condition Procedures

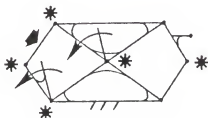
1)



a) Necessary Condition Procedure M for 2456.

b) Necessary Condition Procedure D for 1234.

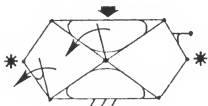
2)



a) Necessary Condition Procedure M for 2456.

b) Necessary Condition Procedure H for 1234.

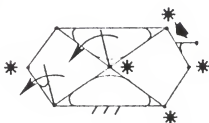
3)



a) Necessary Condition Procedure M for 2456.

b) Necessary Condition Procedure G for 1234.

4)



a) Necessary Condition Procedure N for 2456.

b) Necessary Condition Procedure G for 1234.

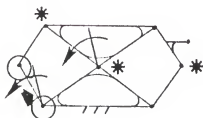
5)



a) Necessary Condition Procedure L for 2456.

b) Necessary Condition Procedure G for 1234.

6)

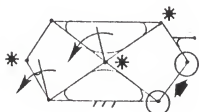


a) Necessary Condition Procedure M for 2456.

b) Necessary Condition Procedure B for 1234.

c) Verify Connection of 56.

7)

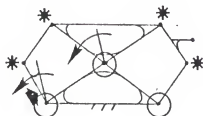


a) Necessary Condition Procedure J for 2456.

b) Necessary Condition Procedure G for 1234.

c) Verify Connection of 13.

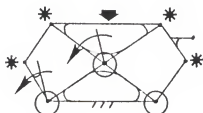
8)



a) Necessary Condition Procedure K for 2456.

b) Necessary Condition Procedure A for 1234.

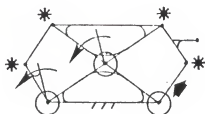
9)



a) Necessary Condition Procedure K for 2456.

b) Necessary Condition Procedure A for 1234.

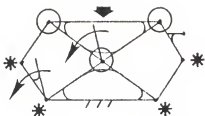
10)



a) Necessary Condition Procedure K for 2456.

b) Necessary Condition Procedure A for 1234.

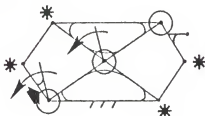
11)



a) Necessary Condition Procedure I for 2456.

b) Necessary Condition Procedure C for 1234.

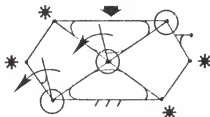
12)



a) Necessary Condition Procedure I for 2456.

b) Necessary Condition Procedure A for 1234.

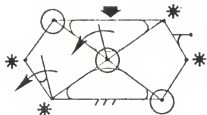
13)



a) Necessary Condition Procedure I for 2456.

b) Necessary Condition Procedure A for 1234.

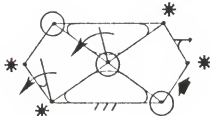
14)



a) Necessary Condition Procedure K for 2456.

b) Necessary Condition Procedure C for 1234.

15)



a) Necessary Condition Procedure K for 2456.

b) Necessary Condition Procedure C for 1234.

APPENDIX C
PROGRAM MODULES

Use of Appendix C

Appendix C is intended to serve as a guide to the program modules needed to perform the analytics presented in this work. Each chart contains a module's functional description and input/output. As such Appendix C could serve as an outline for a program package on nearly any system providing adequate storage. At the present time the package is probably most effectively implemented in interactive APL, and the subroutine calls, "go-tos" and input/output conventions used are those of APL.

Here the flow of data between modules utilizes user-specified files with the restriction that file numbers 1 to 30 are reserved for operating storage. Occasionally a module's specified file arguments are not entirely applicable (usually for four-bar applications using four/six-bar routines), in which case reference should be made to file zero. Once an optimization routine has been created and module I/O coordinated, all human/computer interfacing is interactive and handled by each module independently.

Notation

(x) File number of file containing data x.

→ APL go-to. The outputted global argument of each module is the next line number to be executed.



The links attached to the circled pivot may have complete relative rotation.



A change point cannot be allowed at the pivot.

Module Chart 1

Name - Initialize

Label - INITIAL

Description

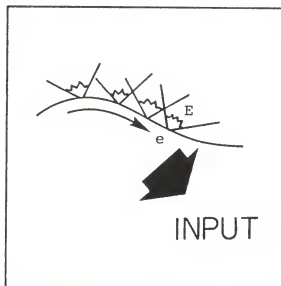
Creates and initializes
files, and collects informa-
tion related to problem type.
First module of every program.

APL Call

"→INITIAL"

Module Chart 2

Name - Synthesis Input
 Label - SYNINP

Description

Interactively collects
 input for synthesis routines.

APL Call

"→(O₁), (O₂) SYNINP"

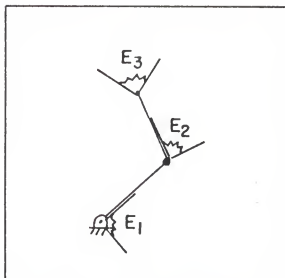
where:

(O₁) = 4MSP set associated with E

(O₂) = Point e. Optional, used if E rotates
 as a crank and e is a prespecified
 pivot.

Module Chart 3

Name - Dyad Type (Synthesis)
 Label - SDYDTYP

Description

Evaluates type for prespecified dyads. Generates a program interrupt prior to synthesis if dyad type is inconsistent for the 4MSP's.

APL Call

"→SDYDTYP (I₁), (I₂), (I₃)"

where:

(I₁) = 4MSP associated with E₁

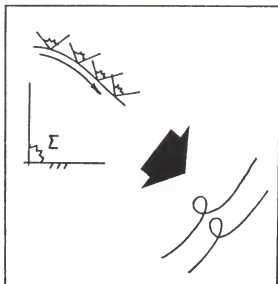
(I₂) = 4MSP associated with E₂

(I₃) = 4MSP associated with E₃

Module Chart 4

Name - Curvature Transform (Synthesis Tool)

Label - SCRVRTN

Description

Performs a curvature transform, generates circle-point and center-point cubics for a given 4MSP.

APL Call

"→(O₁), (O₂), (O₃), (O₄) SCRVRTN (I₁), (I₂), (I₃)"

where:

(I₁) = MSP set.

(I₂), (I₃) = Pivots of predefined constraint link (moving, fixed), if one exists.

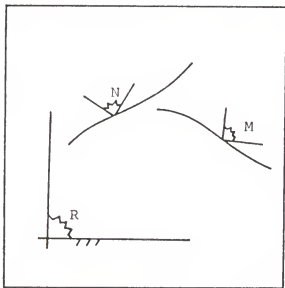
(O₁), (O₂) = Moving, fixed pivots, first constraint link.

(O₃), (O₄) = Moving, fixed pivots, second constraint link (optional for Stephenson dyad links).

Module Chart 5

Name - Inversion (Synthesis Tool)

Label - SINVERT

Description

Generates a new 4MSP set representing a change of reference.

Case #1 - Given N relative to R, M relative to R, generate M relative to N.

Case #2 - Given N relative to R, M relative to N, generate M relative to R.

Case #3 - Given M relative to R, generate R relative to M.

APL Call

"→(O₁) SINVERT (I₁), (I₂), (I₃)"

where:

(I₁) = 4MSP associated with M.

(I₂) = 4MSP associated with N.

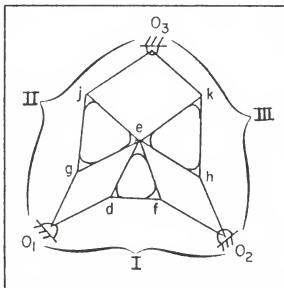
I₃ = 1, 2 or 3, case number

(O₁) = Generated 4MSP set (M relative to
1) N or 2) R, or 3) R relative to N).

Module Chart 6

Name - Path Cognate (Synthesis Tool)

Label - SPTHCOG

Description

Given a four-bar (I),
generates path-cognate four-
bars II and III.

APL Call

"→(O₁), (O₂), (O₃), (O₄) SPTHCOG (I₁), (I₂), (I₃), (I₄), (I₅)"

where:

(I₁) = 4MSP of def (identifies e)

(I₂), (I₃) = link $\overline{dO_1}$ pivots

(I₄), (I₅) = link $\overline{fO_2}$ pivots

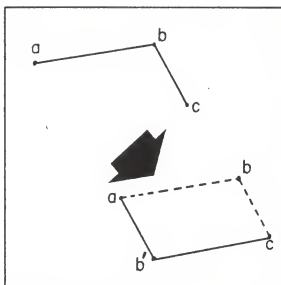
(O₁), (O₂) = $\overline{gO_1}$ (and $\overline{hO_2}$)

(O₃), (O₄) = $\overline{jO_3}$ (and $\overline{kO_4}$)

Module Chart 7

Name - Angular Cognate (Synthesis Tool)

Label - SANGCOG

Description

Generates the angular cognate of dyad abc.

APL Call

"→(O₁) SANGCOG (I₁), (I₂), (I₃)"

where:

(I₁) = pivot a

(I₂) = pivot b

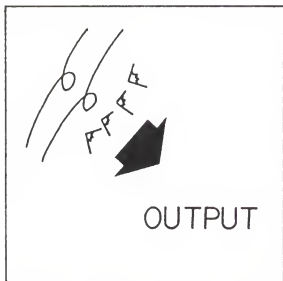
(I₃) = pivot c

(O₁) = pivot b'

Module Chart 8

Name - Synthesis Output

Label - SYNOUT

Description

Displays 4MSP set, generated cubics, focus, Ball point, inverse Ball point, axes.

APL Call

"→SYNOUT (I_1), (I_2), (I_3), (I_4)"

where:

(I_1), (I_2) = Circle, center points of first constraints.

(I_3), (I_4) = Circle, center points of second constraints. (Optional for Stephenson dyad links).

Module Chart 9

Name - Necessary Condition Input
Label - NECINP

Description

Presently has no function, is retained only for reasons of consistency and possible use in future expansion.

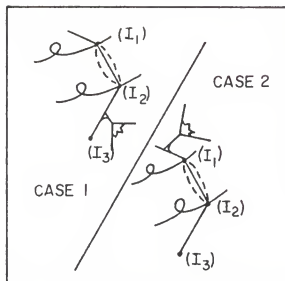
APL Call

"→NECINP"

Module Chart 10

Name - Dyad Type (Necessary Condition)

Label - NDYDTYP

Description

Evaluates dyads with a synthesized link on the basis of consistency of dyad type for the 4MSP. Generates a necessary condition grid (1-D) indicating acceptable dyads.

APL Call

"→ (O₁) NDYDTYP (I₁), (I₂), (I₃), (I₄)"

where:

(I₁), (I₂) = Pivots defining synthesized link.

(I₃) = Pivot defining third dyad pivot.

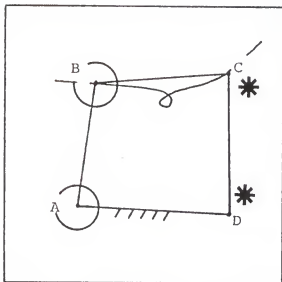
I₄ = 1 or 2, case number.

(O₁) = Output grid.

Module Chart 11

Name - Necessary Condition Procedure A

Label - NCPA

Description

Evaluates four-bar chains on the basis of change-point considerations for the case shown. Creates a necessary condition grid indicating acceptable linkages.

APL Call

"→ (O₁) NCPA (I₁) (I₂) (I₃) (I₄) (I₅)"

where:

(I₁), (I₂) = Pivots of first constraint link
(moving, fixed pivots).

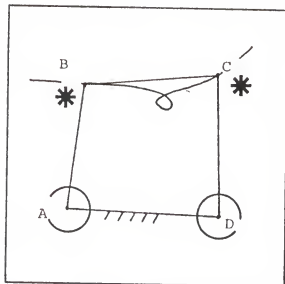
(I₃), (I₄) = Pivots of second constraint link
(moving, fixed pivots).

I₅ = 1 or 2, identifies constraint
link with change-point restrictions.

(O₁) = Output grid.

Module Chart 12

Name - Necessary Condition Procedure B
 Label - NCPB

Description

Evaluates four-bar chains on the basis of change-point considerations for the case shown. Creates a necessary condition grid indicating acceptable linkages.

APL Call

"→(O₁) NCPB (I₁) (I₂) (I₃) (I₄)"

where:

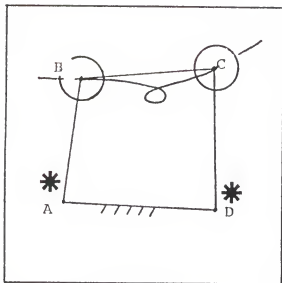
(I₁), (I₂) = Pivots of first constraint link
 (moving, fixed pivots).

(I₃), (I₄) = Pivots of second constraint link
 (moving, fixed pivot).

(O₁) = Output grid.

Module Chart 13

Name - Necessary Condition Procedure C
 Label - NCPC

Description

Evaluates four-bar chains on the basis of change-point considerations for the case shown. Creates a necessary condition grid indicating acceptable linkages.

APL Call

"→(O₁) NCPC (I₁) (I₂) (I₃) (I₄)"

where:

(I₁), (I₂) = Pivots of first constraint link
 (moving, fixed pivots).

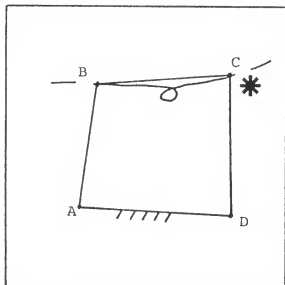
(I₃), (I₄) = Pivots of second constraint link
 (moving, fixed pivot).

(O₁) = Output grid.

Module Chart 14

Name - Necessary Condition Procedure D

Label - NCPD

Description

Evaluates four-bar chains on the basis of change-point considerations for the case shown. Creates a necessary condition grid indicating acceptable linkages.

APL Call

"→(O₁) NCPD (I₁) (I₂) (I₃) (I₄) (I₅)"

where:

(I₁), (I₂) = Pivots of first constraint link
(moving, fixed pivots).

(I₃), (I₄) = Pivots of second constraint link
(moving, fixed pivots).

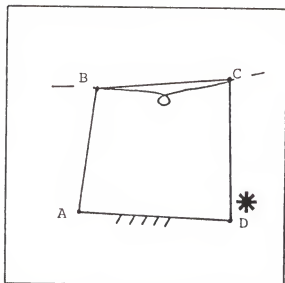
I₅ = 1 or 2, identifies constraint
link with change-point restrictions.

(O₁) = Output grid.

Module Chart 15

Name - Necessary Condition Procedure G

Label - NCPG

Description

Evaluates four-bar chains on the basis of change-point considerations for the case shown. Creates a necessary condition grid indicating acceptable linkages.

APL Call

"→ (O₁) NCPG (I₁) (I₂) (I₃) (I₄) (I₅)"

where:

(I₁), (I₂) = Pivots of first constraint link
(moving, fixed pivots).

(I₃), (I₄) = Pivots of second constraint link
(moving, fixed pivots).

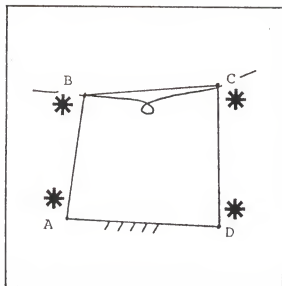
I₅ = 1 or 2, identifies constraint
link with change-point restrictions.

(O₁) = Output grid.

Module Chart 16

Name - Necessary Condition Procedure II

Label - NCPH

Description

Evaluates four-bar chains on the basis of change-point considerations for the case shown. Creates a necessary condition grid indicating acceptable linkages.

APL Call

$$\rightarrow (O_1) \text{ NCPH } (I_1) (I_2) (I_3) (I_4) "$$

where:

$(I_1), (I_2)$ = Pivots of first constraint link
(moving, fixed pivots).

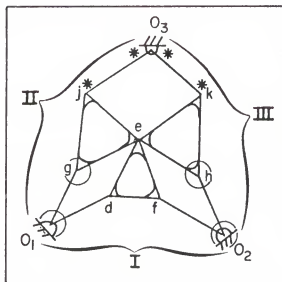
$(I_3), (I_4)$ = Pivots of second constraint link
(moving, fixed pivot).

(O_1) = Output grid.

Module Chart 17

Name - Necessary Condition Procedure I

Label - NCPI

Description

Evaluates synthesized four-bar I on the basis of change-point considerations within path cognate four-bars II and III as shown. Generates a necessary condition grid (2x2-D) indicating acceptable cognates.

APL Call

"→(O₁) NCPI (I₁), (I₂), ... (I₈)"

where:

(I₁), (I₂) = link $\overline{dO_1}$

(I₃), (I₄) = link $\overline{fO_2}$

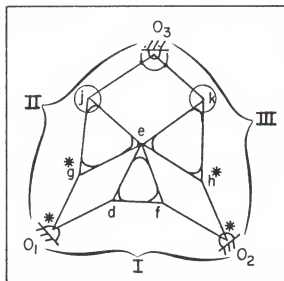
(I₅), (I₆) = link $\overline{gO_1}$ (and $\overline{hO_2}$)

(I₇), (I₈) = link $\overline{jO_3}$ (and $\overline{kO_3}$)

(O₁) = Output grid.

Module Chart 18

Name - Necessary Condition Procedure J
 Label - NCPJ

Description

Evaluates synthesized four-bar I on the basis of change-point considerations within path cognate four-bars II and III as shown. Generates a necessary condition grid (2x2-D) indicating acceptable cognates.

APL Call

"→(O₁) NCPJ (I₁), (I₂), ... (I₈)"

where:

(I₁), (I₂) = link $\overline{dO_1}$

(I₃), (I₄) = link $\overline{fO_2}$

(I₅), (I₆) = link $\overline{gO_1}$ (and $\overline{hO_2}$)

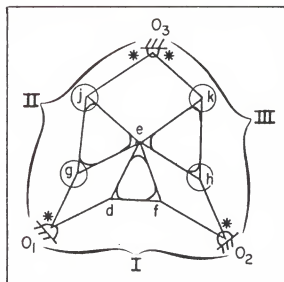
(I₇), (I₈) = link $\overline{jO_3}$ (and $\overline{kO_3}$)

(O₁) = Output grid.

Module Chart 19

Name - Necessary Condition Procedure K

Label - NCPK

Description

Evaluates synthesized four-bar I on the basis of change-point considerations within path cognate four-bars II and III as shown. Generates a necessary condition grid (2x2-D) indicating acceptable cognates.

APL Call

"→(O₁) NCPK (I₁), (I₂), ... (I₈)"

where:

(I₁), (I₂) = link $\overline{dO_1}$

(I₃), (I₄) = link $\overline{fO_2}$

(I₅), (I₆) = link $\overline{gO_1}$ (and $\overline{hO_2}$)

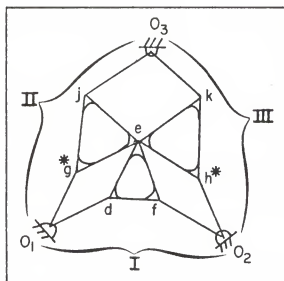
(I₇), (I₈) = link $\overline{jO_3}$ (and $\overline{kO_3}$)

(O₁) = Output grid.

Module Chart 20

Name - Necessary Condition Procedure L

Label - NCPL

Description

Evaluates synthesized four-bar I on the basis of change-point considerations within path cognate four-bars II and III as shown. Generates a necessary condition grid (2x2-D) indicating acceptable cognates.

APL Call

"→(O₁) NCPL (I₁), (I₂), ... (I₈)"

where:

(I₁), (I₂) = link $\overline{dO_1}$

(I₃), (I₄) = link $\overline{fO_2}$

(I₅), (I₆) = link $\overline{gO_1}$ (and $\overline{hO_2}$)

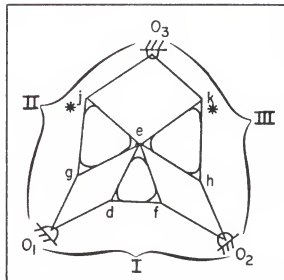
(I₇), (I₈) = link $\overline{jO_3}$ (and $\overline{kO_3}$)

(O₁) = Output grid.

Module Chart 21

Name	Necessary Condition	Procedure	M

Label - NCPM



Description

Evaluates synthesized four-bar I on the basis of change-point considerations within path cognate four-bars II and III as shown. Generates a necessary condition grid (2x2-D) indicating acceptable cognates.

APL Call

" $\rightarrow (O_1)$ NCPM $(I_1), (I_2), \dots, (I_8)$ "

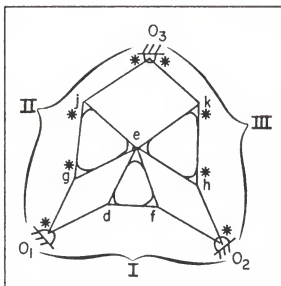
where:

$$(I_1), (I_2) = \text{link } \overline{dO_1}$$
$$(I_3), (I_4) = \text{link } \overline{fO_2}$$
$$(I_5), (I_6) = \text{link } \overline{gO_1} \text{ (and } \overline{hO_2})$$
$$(I_7), (I_8) = \text{link } \overline{JO_3} \quad (\text{and } \overline{KO_3})$$

(O₁) = Output grid.

Module Chart 22

Name - Necessary Condition Procedure N
 Label - NCPN

Description

Evaluates synthesized four-bar I on the basis of change-point considerations within path cognate four-bars II and III as shown. Generates a necessary condition grid (2x2-D) indicating acceptable cognates.

APL Call

"→(O₁) NCPN (I₁), (I₂), ... (I₈)"

where:

(I₁), (I₂) = link $\overline{dO_1}$

(I₃), (I₄) = link $\overline{fO_2}$

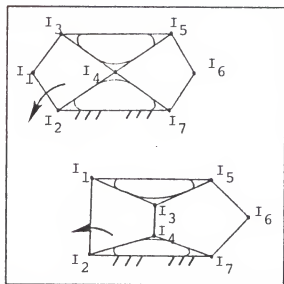
(I₅), (I₆) = link $\overline{gO_1}$ (and $\overline{hO_2}$)

(I₇), (I₈) = link $\overline{jO_3}$ (and $\overline{kO_4}$)

(O₁) = Output grid.

Module Chart 23

Name - Necessary Condition Dyad Analysis
Label - NECDYAD

Description

Performs a dyad-based analysis, evaluates the solution set of linkages on the basis of:

- Connection
- Disconnection
- Encirclement
- Non-encirclement
- g-function reversal

APL Call

"→(O₁) NECDYAD (I₁), (I₂), ..."

where:

For six-bars, Stephenson 3 or Watt 2 inversion.

(I ₁), (I ₂) = Crank pivots (moving, fixed)
(I ₃), (I ₄) = Follower pivots (moving, fixed)
(I ₅), (I ₆), (I ₇) = Dyad pivots (=0 for four-bar)
(I ₈) = MSP set, coupler or crank relative to ground. (Can be 0 if I ₉ = 1)

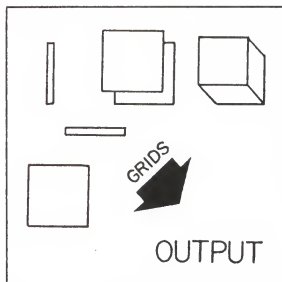
I₉ = Mode of drive.
 = 1 for cyclic drive.
 = 2 from position 0 to 3.
 = 3 through range of disconnection.

I₁₀⁺ = Necessary conditions
 = 1 for Connection
 = 2 for Disconnection ($\overline{I_6 I_7}$ short)
 = 3 for Disconnection ($\overline{I_5 I_6}$ short)
 = 4 for Encirclement
 = 5 for Non-encirclement.
 = 6 for g-function reversal (Ground relative to $\overline{I_6 I_7}$).
 = 7 for g-function reversal ($\overline{I_5 I_6}$ relative to $\overline{I_6 I_7}$).

(O₁) = Output grid.

Module Chart 24

Name - Necessary Condition Output
 Label - NECOUT

Description

Displays grids generated by previous necessary condition modules. Collects expansion input for the selected portion of the grid.

APL Call

"→NECOUT (I_1), (I_2), ..."

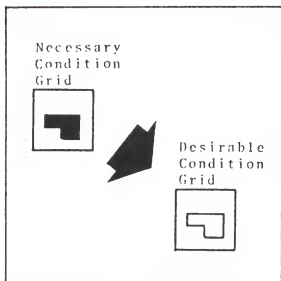
where:

(I_1), (I_2) = grids to be displayed.

Module Chart 25

Name - Desirable Condition Input

Label - DESINP

Description

Given a grid of acceptable linkages from the necessary conditions, a scoring grid is initialized and placed in files.

APL Call

$$"+(O_1) \text{ DESINP } (I_1), (I_2) \dots"$$

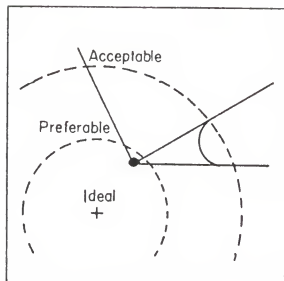
where:

$(I_1), (I_2) \dots$ = Necessary condition grids,
(If more than one, combines them).

(O_1) = Initialized scoring grid for desirable conditions.

Module Chart 26

Name - Pivot Location (Desirable Condition)
 Label - DPVTLCA

Description

Evaluates the solution set of linkages on the basis of pivot location. User specifies pivots of interest.

APL Call

"→ (O₁) (O₂) DPVTLCA (I₁), (I₂)"

where:

	(I ₁) = Previous scoring grid
	(I ₂) = A pivot
	(I ₃) = B pivot
	(I ₄) = C pivot
	(I ₅) = D pivot
Six bars only	(I ₆) = F pivot
	(I ₇) = G pivot
	(I ₈) = H pivot

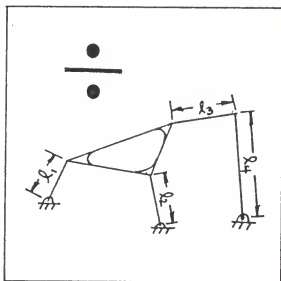
(O₁) = Output scoring grid, cumulative.

(O₂) = Output scoring grid, this condition only. Optional.

Module Chart 27

Name - Link length Ratio (Desirable Condition)

Label - DLKLNRT

Description

Evaluates the solution set of linkages on the basis of link length ratio. User specifies links of interest.

APL Call

"→(O₃) (O₂) DLKNRT (I₁), (I₂), (I₃), ..."

where:

(I₁) = Previous scoring grid.

(I₁), (I₃); (I₄), (I₅); ... = Constraint link
pivots (1 pair =
1 link).

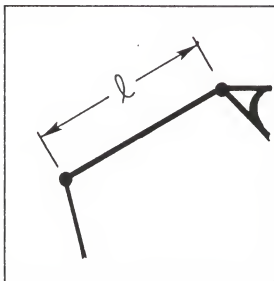
(O₁) = Output scoring grid, cumulative.

(O₂) = Output scoring grid, this condition
only. Optional.

Module Chart 28

Name - Link Size (Desirable Condition)

Label - DLKSIZE

Description

Evaluates the solution set of linkages on the basis of link size. User specifies links of interest.

APL Call

"→(O₁), (O₂) DLKSIZE (I₁) (I₂) (I₃)"

where:

(I₁) = Previous scoring grid.

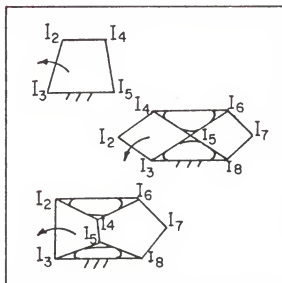
(I₂), (I₃); (I₄) (I₅); ... = Constraint link
Pivots (1 pair =
1 link)

(O₁) = Output scoring grid, cumulative.

(O₂) = Output scoring grid, this condition
only. Optional.

Module Chart 29

Name - Desirable Condition Dyad Analysis
 Label - DESDYAD

Description

Performs a dyad-based analysis, evaluates the solution set of linkages on the basis of:

- Overall Boundary
- g-function
- Δh
- hrms
- Δ Torque
- Shock Level

APL Call

" $\rightarrow (O_1), (O_2) \dots \text{DESDYAD } (I_1), (I_2), \dots$ "

where:

for six bars,
 Stephenson 3
 of Watt 2
 inversion.

- (I_1) = Previous scoring grid
- $(I_2)(I_3)$ = Crank pivots (moving, fixed)
- $(I_4)(I_5)$ = Follower pivots (moving, fixed)
- $(I_6)(I_7)(I_8)$ = Dyad pivots (= 0 for four-bar)
- (I_9) = MSP set, coupler or crank relative to ground (may be 0 if $I_{10} = 1$)
- I_{10} = Mode of drive.
 - = 1 for cyclic drive.
 - = 2 from position 0 to 3.
 - = 3 through range of disconnection.
- I_{11}^+ = Desirable conditions
 - = 1 for Overall boundary
 - = 2 for g-function
 - = 3 for Δh -function
 - = 4 for hrms-function
 - = 5 for Δ Torque
 - = 6 for Shock Level

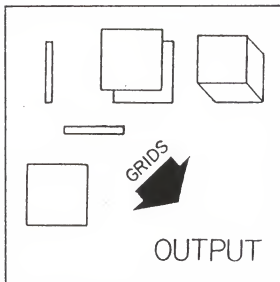
(O₁) = Output scoring grid, cumulative
(all desirable conditions).

(O₂), (O₃) ... = Output scoring grids, separate conditions. Optional.

Module Chart 30

Name - Desirable Condition Output

Label - DESOUT

Description

Outputs scoring grid from desirable condition analysis. Collects expansion input for the selected portion of the grid.

APL Call

"→DESOUT (I_1), (I_2), ..."

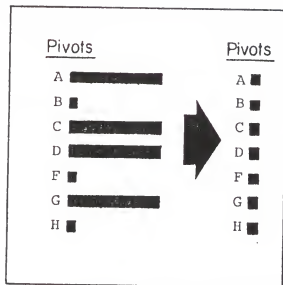
where:

(I_1), (I_2).... = Scoring grids to be displayed.
(If more than one, add them).

Module Chart 31

Name - Extract

Label - EXTRACT

Description

Allows user to select a particular linkage from a grid; compresses pivot files to represent only the selected linkage. Useful for dependent synthesis problems.

APL Call

"→EXTRACT (I_1)"

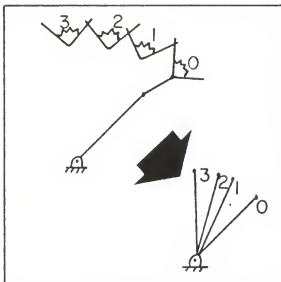
where:

(I_1) = Scoring grid (previously outputted)

Module Chart 32

Name - Analyze

Label - ANALYZE

Description

Given a crank in zeroth position, 4MSP of coupler, generates 4MSP of rotating crank.

APL Call

" $\rightarrow (O_1)$ ANALYZE $(I_1), (I_2), (I_3)$ "

where:

$(I_1), (I_2)$ = moving, fixed pivots of crank in zeroth position.

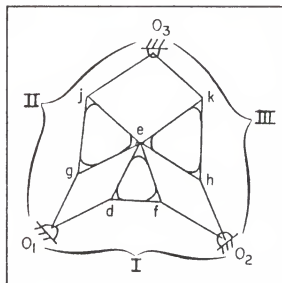
(I_3) = 4MSP of coupler.

(O_1) = 4MSP of crank.

Module Chart 33

Name - Analyze Path-Cognate

Label - ANLZPCG

Description

Given 4MSP of coupler of a synthesized four-bar and pivot locations in the zeroth position of one of its path cognates, generates (3) sets of 4MSP representing motion of path cognate's links.

APL Call

"→ (O₁), (O₂), (O₃) ANLZPCG (I₁), (I₂), (I₃), (I₄), (I₅)"

where:

(I₁) = 4MSP of coupler of four-bar O₁dfO₂

(I₂) = g in zeroth position.

(I₃) = O₁ in zeroth position.

(I₄) = j in zeroth position.

(I₅) = O₃ in zeroth position.

(O₁) = 4MSP of $\overline{gO_1}$

(O₂) = 4MSP of $\overline{jg}$

(O₃) = 4MSP of $\overline{jO_3}$

APPENDIX D
SAMPLE PROGRAMS

Each chart in Appendix D contains a sample program demonstrating a coordination of the modules of Appendix C to create an optimization routine for a particular problem/linkage/mode of cyclicity set. There is an example for each of the twelve problem types, each one using an Appendix B solution.

Three types of data are channeled between modules through files. They are motion specification sets, pivot coordinates and grids. The nomenclature used to represent these quantities is explained below.

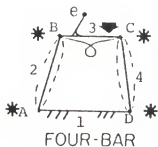
Notation

"4MSP (3/4) "	Motion specification set of four multiply separated positions describing the motion of plane 3 relative to plane 4.
"pivot A"	The coordinates of pivot A. May represent a set of pivots or a single pivot.
"pivot A'"	In path-cognate problems, the pivots generated by the curvature transform are denoted by a prime ('), and cognate pivots (representing solutions) are unprimed.

- NC Grid (3) Necessary condition grid for a set of #3 links (1-dimensional).
- DC Grid (3,5) Desirable condition grid for sets of #3 and #5 links (2-dimensional).
- "36+17" "17" in APL represents 1 2 3 4 5 6 7.
36+17 represents 37 38 39 40 41 42 43.
- "→" APL go-to. Each module generates as an argument the next line to be executed.
-  Driven link.
-  Cyclic pivot. Attached links have complete relative rotation.
-  A change point cannot be allowed at this pivot.
-  Encircled link is synthesized using the curvature transform (circle point cubic shown).

Program Chart 1

Problem #1 - Path Synthesis.
Linkage Solution - Four-Bar
Mode of Cyclicity #2

Synthesis Procedure

- 1) CURVATURE TRANSFORM to find 2,4.

Necessary Condition Procedure

- 1) Necessary Condition Procedure H for 1234.

Sample Program

VPROB1FBMODF2

```
[1] → INITIAL
[2] → 31 SYNINP
[3] → 35 32 34 33 SCRVRTN 31
[4] → SYNOUT 35 32 34 33

[5] → NECINP
[6] → 36 NCPH 35 32 34 33
[7] → NECOUT 36

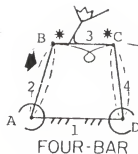
[8] → 37 DESINP 36
[9] → 37 DPVTLC 37 32 35 34 33
[10] → 37 DLKLNRT 37 33 34 34 35 35 32
[11] → 37 DLKSIZE 37 35 34
[12] → 37 DESDYAD 37 35 32 34 33 0 0 0 31
      2, 16
[13] → DESOUT 37V
```

Files

```
31) 4MSP(3/1)
32) pivot A
33) pivot D
34) pivot C
35) pivot B
36) NC Grid(4x2)
37) DC Grid(4x2)
```

Program Chart 2

Problem #2 - Motion Synthesis,
Linkage Solution - Four-Bar
Mode of Cyclicity #2

Synthesis Procedure

- 1) CURVATURE TRANSFORM to find 2, 4.

Necessary Condition Procedure

- 1) Necessary Condition Procedure B for 1234.

Sample Problem

VPROB2FBMODE4

```

[1] → INITIAL
[2] → 31 SYNINP
[3] → 35 32 34 33 SCRVRTN 31
[4] → SYNOUT 35 32 34 33

[5] → NECINP
[6] → 36 NCPB 35 32 34 33
[7] → NECOUT 36

[8] → 37 DESINP 36
[9] → 37 DPVTLCA 37 32 35 34 33
[10] → 37 DLKLNRT 37 34 33 35 32
[11] → 37 DLKSIZE 37 32 33 35 34
[12] → 37 DESDYAD 37 35 32 34 33 0 0 0
      31 1, 16
[13] → DESOUT 37V
  
```

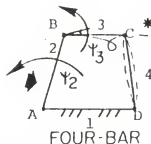
Files

```

31) 4MSP(3/1)
32) pivot A
33) pivot D
34) pivot C
35) pivot B
36) NC Grid(4x2)
37) DC Grid(4x2)
  
```

Program Chart 3

Problem #3 - Angular Coordination
of a Two-Link Chain.
Solution Linkage - Four-Bar
Mode of Cyclicity #1

Synthesis Procedure

- 1) CURVATURE TRANSFORM to find 4.

Necessary Condition Procedure

- 1) Necessary Condition Procedure D for 1234.

Sample Program

VPROB3FBMODEL

```
[1] → INITIAL
[2] → 31 33 SYNINP
[3] → 32 36 SYNINP
[4] → 36 33 35 34 SCRVRTN 32 36 33
[5] → SYNOUT 35 34 36 33

[6] → NECINP
[7] → 37 NCPD 35 34 36 33 1
[8] → NECOUT 37

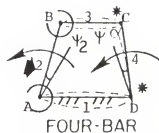
[9] → 38 DESINP 37
[10] → 38 DPVTLCA 38 33 36 35 34
[11] → 38 DLKLNRT 38 35 34 36 33
[12] → 38 DLKSIZE 38 34 33
[13] → 38 DESDYAD 38 36 33 35 34,
      0 0 0 31 2, 16
[14] → DESOUT 38V
```

Files

```
31) 4MSP(2/1)
32) 4MSP(3/1)
33) pivot A
34) pivot D
35) pivot C
36) pivot B
37) NC Grid (4)
38) DC Grid (4)
```

Program Chart 4

Problem #4 - Angular Coordination
of Two Cranks.
Linkage Solution - Four-Bar
Mode of Cyclicity #3

Synthesis Procedure

- 1) INVERSION to 2.
- 2) CURVATURE TRANSFORM to find 3.

Necessary Condition Procedure

- 1) Necessary Condition Procedure B for 1234.

Sample Problem

VPROB4FBMODE3

```

[1] → INITIAL
[2] → 31 33 SYNINP
[3] → 32 34 SYNINP
[4] → 37 SINVERT 32 31 1
[5] → 34 33 35 36 SCRVRTN 37
[6] → SYNOUT 34 33 35 36

[7] → NECINP
[8] → 38 NCPB 34 33 35 36
[9] → NECOUT 38

[10] → 39 DESINP 38
[11] → 39 DPVTLCA 39 33 36 35 34
[12] → 39 DLKLNRT 39 33 34 34 35 35 36 36 33
[13] → 39 DLKSIZE 39 33 36
[14] → 39 DESDYAD 39 36 33 35 34 0 0 0 0
      1, 16
[15] → DESOUT 39V
  
```

Files

```

31) 4MSP(2/1)
32) 4MSP(4/1)
33) pivot A
34) pivot D
35) pivot C
36) pivot B
37) 4MSP(4/2)
38) NC Grid(1x3)
39) DC Grid(1x3)
  
```

Program Chart 5

Problem #5 - Path Coordination
with a Crank.
Linkage Solution - Four-Bar
Mode of Cyclicity - #5

Synthesis Procedure

- 1) CURVATURE TRANSFORM and
PATH COGNATES to find
2, 4.

Necessary Condition Procedure

- 1) Necessary Condition Pro-
cedure J for 1234.

Sample Program

VPROB5FBMODE5

```

[1] → INITIAL
[2] → 31 SYNINP
[3] → 32 33 34 35 SCRVTN 31
[4] → 36 37 38 39 SPTHCOG 31 32 33 34 35
[5] → NECINP
[6] → 40 NCPJ 31+18
[7] → NECOUT 40
[8] → 41 DESINP 40
[9] → 41 DPVTLCA 41 37 36 38 39
[10] → 41 DLKLNRT 41 36 38 36 37
[11] → 41 DLKSIZE 41 38 39 37 39
[12] → 41 DESDYAD 41 38 39 36 37 0 0 0 0
      1, 16
[13] → DESOUT 41 V

```

Files

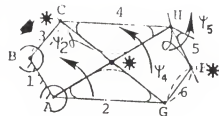
```

31) 4MSP(3/1)
32) pivot B'
33) pivot A'
34) pivot C'
35) pivot D'
36) pivot B
37) pivot A
38) pivot C
39) pivot D
40) NC Grid(2x4)
41) DC Grid(2x4)

```

Program Chart 6

Problem #6 - Angular Coordination
of a Three Link Chain.
Linkage Solution - Watt 1
Mode of Cyclicity #8



WATT 1

Synthesis Procedure

- 1) CURVATURE TRANSFORM to find 3.
- 2) INVERSION to 2.
- 3) CURVATURE TRANSFORM to find 6.

Necessary Condition Procedure

- 1) Necessary Condition Procedure B for 1234.
- 2) Necessary Condition Procedure D for 2456.
- 3) Verify connection of 56.

Sample Program

VPROB6WATT1MODE8

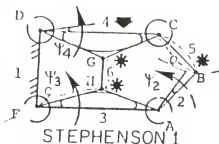
```
[1] → INITIAL
[2] → 31 34 SYNINP
[3] → 32 40 SYNINP
[4] → 33 37 SYNINP
[5] → 40 34 38 39 SCRVRTN 32 40 34
[6] → 41 SINVERT 33 31 1
[7] → 42 SINVERT 0 31 3
[8] → 37 40 36 35 SCRVRTN 41 37 40
[9] → SYNOUT 38 39 40 34
[10] → SYNOUT 36 35 37 40
[11] → NECINP
[12] → 43 NCPB 38 39 40 34
[13] → 44 NCPD 36 35 37 40 1
[14] → 45 NECDYAD 39 34 38 40 37
      36 35 42 1 1
[15] → NECOUT 45
[16] → 46 DESINP 45
[17] → 46 DPVTICA 46 34 39 38 40 36 35 37
[18] → 46 DLKLNRT 46 37 36 38 39
[19] → 46 DLKSIZE 46 38 40
[20] → 46 DESDYAD 46 39 34 38 40
      37 36 35 42 1, 16
[21] → DESOUT 46 v
```

Files

- 31) 4MSP (2/1)
- 32) 4MSP (4/1)
- 33) 4MSP (5/1)
- 34) pivot A
- 35) pivot G
- 36) pivot F
- 37) pivot H
- 38) pivot C
- 39) pivot B
- 40) pivot D
- 41) 4MSP (5/2)
- 42) 4MSP (1/2)
- 43) NC Grid (3)
- 44) NC GRID (6)
- 45) NC Grid (3,6)
- 46) DC Grid (3,6)

Program Chart 7

Problem #7 - Angular Coordination
of a Crank and a Two-Link Chain.
Linkage Solution - Stephenson 1
Mode of Cyclicity #9

Synthesis Procedure

- 1) INVERSION to 4.
- 2) CURVATURE TRANSFORM to find 6.
- 3) CURVATURE TRANSFORM to find 5.

Necessary Condition Procedure

- 1) Necessary Condition Dyad Type (25).
- 2) Necessary Condition Procedure A for 1346
- 3) Verify Connection of 25.
- 4) Verify Encirclement (AC).

Sample Program

VPROB7STEPH1MODE9

```
[1] → INITIAL

[2] → 31 34 SYNINP
[3] → 32 35 SYNINP
[4] → 33 38 SYNINP
[5] → 41 SINVERT 33 31 1
[6] → 42 SINVERT 33 32 1
[7] → 34 38 40 38 SCRVRTN 41 34 38
[8] → 36 37 SCRVRTN 42
[9] → SYNOUT 40 39 34 38
[10] → SYNOUT 36 37

[11] → NECINP
[12] → 44 NDYDTYP 37 36 35 1
[13] → 43 NCFA 40 39 34 38 1
[14] → 45 NECDYAD 34 38 40 39 35,
      36 37 41 1 1 4
[15] → NECOUT 45

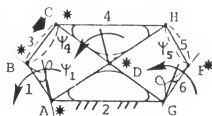
[16] → 46 DESINP 45
[17] → 46 DPVTICA 46 35 36 37 38 34 39 40
[18] → 46 47 DLKLNRT 45 36 37 36 35
[19] → 46 DLKSIZE 46 35 34 37 38
[20] → 46 DESDYAD 46 34 38 40 39,
      35 36 37 41 1,16
[21] → DESOUT 46 V
```

Files

```
31) 4MSP (3/1)
32) 4MSP (2/1)
33) 4MSP (4/1)
34) pivot F
35) pivot A
36) pivot B
37) pivot C
38) pivot D
39) pivot G
40) pivot H
41) 4MSP (3/4)
42) 4MSP (2/4)
43) NC Grid (6)
44) NCGrid (5)
45) NC Grid (6,5)
46) DC Grid (6,5)
47) DC Grid (LLR
    only)
```

Program Chart 8

Problem #8 - Angular Coordination
of Three Cranks.
Linkage Solution - Watt 2
Mode of Cyclicity #2



WATT 2

Synthesis Procedure

- 1) INVERSION to 4.
- 2) CURVATURE TRANSFORM to find 3.
- 3) CURVATURE TRANSFORM to find 5.

Necessary Condition Procedure

- 1) Necessary Condition Procedure H for 1234.
- 2) Necessary Condition Procedure D for 2456.

Sample Program

VPROB8WATT2MODE2

```

[1] → INITIAL
[2] → 31 34 SYNINP
[3] → 32 40 SYNINP
[4] → 33 35 SYNINP
[5] → 41 SINVERT 31 32 1
[6] → 42 SINVERT 33 32 1
[7] → 34 40 39 38 SCRVRTN 41 34 40
[8] → 35 40 36 37 SCRVRTN 42 35 40
[9] → SYNOUT 39 38 34 40
[10] → SYNOUT 36 37 35 40

[11] → NECINP
[12] → 43 NCPH 39 38 34 40
[13] → 44 NCPD 36 37 35 40
[14] → NECOUT 43 44

[15] → 45 DESINP 43 44
[16] → 45 DPVTICA 45 34 39 38 40 36 35 37
[17] → 45 DLKLNRT 45 37 40 38 40
[18] → 45 DLKSIZE 45 37 38
[19] → 45 DESDYAD 45 39 34 38 40 37 36 35
      31 2, 16
[20] → DESOUT 45 V

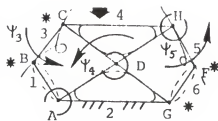
```

Files

- 31) 4MSP (1/2)
- 32) 4MSP (4/2)
- 33) 4MSP (6/2)
- 34) pivot A
- 35) pivot G
- 36) pivot F
- 37) pivot H
- 38) pivot C
- 39) pivot B
- 40) pivot D
- 41) 4MSP (1/4)
- 42) 4MSP (6/4)
- 43) NC Grid (3)
- 44) NC Grid (5)
- 45) DC Grid (3,5)

Program Chart 9

Problem #9 - Angular Coordination
of Three Adjacent Planes.
Linkage Solution - Watt 2
Mode of Cyclicity #9



WATT 2

Synthesis Procedure

- 1) CURVATURE TRANSFORM to find 1.
- 2) CURVATURE TRANSFORM to find 6.

Necessary Condition Procedure

- 1) Necessary Condition Procedure B for 1234.
- 2) Necessary Condition Procedure A for 2456.

Sample Program

VPROB9WATT2MODE9

```

[1] → INITIAL
[2] → 31 40 SYNINP
[3] → 32 38 SYNINP
[4] → 33 37 SYNINP
[5] → 38 40 34 39 SCRVRTN 32 38 40
[6] → 37 40 36 35 SCRVRTN 33 37 40
[7] → SYNOUT 34 39 38 40
[8] → SYNOUT 36 35 37 40

[9] → NECINP
[10] → 41 NCFB 34 39 38 40
[11] → 42 NCPA 36 35 37 40
[12] → NECOUT 41 42

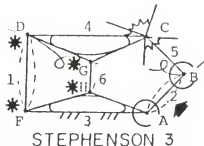
[13] → 43 DESINP 41 42
[14] → 43 DPTLCA 43 34 35 36 37 38
[15] → 43 DKLNRRT 43 37 36 34 39
[16] → 43 DLKSIZE 43 38 40 37 40 39 35
[17] → 43 DESDYAD 43 34 39 38 40 37 36
           35 0 1, 16
[18] → DESOUT 43 "
```

Files

- 31) 4MSP (4/2)
- 32) 4MSP (3/2)
- 33) 4MSP (5/2)
- 34) pivot A
- 35) pivot G
- 36) pivot F
- 37) pivot H
- 38) pivot C
- 39) pivot B
- 40) pivot D
- 41) NC Grid (1)
- 42) NC Grid (6)
- 43) DC Grid (1,6)

Program Chart 10

Problem #10 - Double Coplanar Synthesis of Two Adjacent Planes.
 Linkage Solution - Stephenson 3
 Mode of Cyclicity #5

Synthesis Procedure

- 1) CURVATURE TRANSFORM to find 1,6.
- 2) CURVATURE TRANSFORM to find 2.

Necessary Condition Procedure

- 1) Necessary Condition Procedure H for 1346.
- 2) Verify Disconnection of 25 (2 short)

Sample Program

VPROB10STEPH3MODE5

```

[1] → INITIAL
[2] → 31 SYNINP
[3] → 32 SYNINP
[4] → 37 33 38 39 SCRVRTN 31
[5] → 35 34 SCRVRTN 31
[6] → SYNOUT 37 33 38 39
[7] → SYNOUT 35 34

[8] → NECINP
[9] → 40 NCPH 37 33 38 39
[10] → 41 NECDYAD 37 33 38 39 36
      35 34 31 2 2
[11] → NECOUT 41

[12] → 42 DESINP 41
[13] → 42 DPVTICA 42 34 35 36 37 33 38 39
[14] → 42 DLKLNRT 42 36 35 38 39
[15] → 42 DLKSIZE 42 37 38 38 36
[16] → 42 DESDYAD 42 37 33 38 39 36
      35 34 31 2, 16
[17] → DESOUT 42 V
  
```

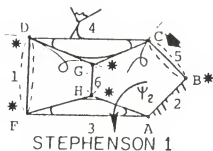
Files

```

31) 4MSP (4/3)
32) 4MSP (5/3)
33) pivot F
34) pivot A
35) pivot B
36) pivot C
37) pivot D
38) pivot G
39) pivot H
40) NC Grid (1,6)
41) NC Grid(1,6,2)
42) DC Grid(1,6,2)
  
```

Program Chart 11

Problem #11 - Coplanar Motion
Coordinated With a Crank.
Linkage Solution - Stephenson 2
Mode of Cyclicity #4



<u>Synthesis Procedure</u>	<u>Necessary Condition Procedure</u>
1) CURVATURE TRANSFORM to find 5. 2) INVERSION to 3. 3) CURVATURE TRANSFORM to find 1,6.	1) Necessary Condition Dyad Type (25). 2) Necessary Condition Procedure II for 1346. 3) Verify Connection of 25. 4) Verify non-g-function Reversal (5/2).

Sample Program

VPROB11STEPH2MODE4

```

[1] → INITIAL
[2] → 31 SYNINP
[3] → 32 34 SYNINP
[4] → 36 35 SCRVRTN 31
[5] → 40 SINVERT 32 31 1
[6] → 37 33 38 39 SCRVRTN 40
[7] → SYNOUT 36 35
[8] → SYNOUT 37 33 38 39

[9] → NECINP
[10] → 41 NDYDTYP 36 35 34 2
[11] → 42 NCPH 37 33 38 39
[12] → 43 NECDYAD 37 33 38 39 36
      35 34 40 2 1 7
[13] → NECOUT 43

[14] → 44 DESINP 43
[15] → 44 DPVTICA 44 34 35 36 37 33 38 39
[16] → 44 DLKLNRT 44 36 38 38 39 39 34
[17] → 44 DLKSIZE 44 37 36 33 34
[18] → 44 DESDYAD 44 37 33 38 39 36 35
      34 40 2, 16
[19] → 44 DESOUT 44 V

```

Files

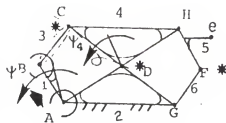
```

31) 4MSP (4/2)
32) 4MSP (3/2)
33) Pivot F
34) pivot A
35) pivot B
36) pivot C
37) pivot D
38) pivot G
39) pivot H
40) 4MSP (4/3)
41) NC Grid (5)
42) NC Grid (1,6)
43) NC Grid (1,6,5)
44) DC Grid (1,6,5)

```

Program Chart 12

Problem #12 - Path Coordination
with Two Cranks.
Linkage Solution - Watt 2
Mode of Cyclicity #6



WATT 2

Synthesis Procedure

- 1) CURVATURE TRANSFORM and PATH COGNATES to find 4,6.
- 2) INVERSION to 1.
- 3) CURVATURE TRANSFORM to find 3.

Necessary Condition Procedure

- 1) Necessary Condition Procedure M for 2456.
- 2) Necessary Condition Procedure B for 1234.
- 3) Verify Connection of 56.

Sample Program

VPROB7WATT2MODE6

```
[1] → INITIAL
[2] → 31 SYNINP
[3] → 32 33 34 35 SCRVRTN 31
[4] → 36 37 38 39 SPTHCOG 31 32 33 34 35
[5] → NECINP
[6] → 40 NCPM 31+18
[7] → NECOUT 40
[8] → 41 DESINP 40
[9] → 41 DPVTLC A 41 37 39 38 36
[10] → 41 DLKLNRT 41 36 38 36 37
[11] → 41 DLKSIZE 41 36 38
[12] → 41 DESDYAD 41 36 37 38 39
      0 0 0 2, 16
[13] → DESOUT 41
[14] → EXTRACT 41
[15] → 45 0 0 ANLZPCG 31 36 37 38 39
[16] → 44 49 SYNINP
```

Files

```
31) 4MSP (4/2)
32) pivot H'
33) pivot D'
34) pivot F'
35) pivot G'
36) pivot H
37) pivot D
38) pivot F
39) pivot G
40) NC Grid (4,6)
41) DC Grid (4,6)
42)
43)
44) 4MSP (1/2)
45) 4MSP (4/2)
46) 4MSP (4/1)
47) pivot C
48) pivot B
49) pivot A
50) NC Grid (3)
51) NC Grid (3,5)
52) DC Grid (3,5)
```

```
[17]→ 46 SINVERT 45 44 1
[18]→ 37 49 47 48 SCRVRTN 46 37 49
[19]→ SYNOUT 47 48 37 49
```

```
[20]→ NECIIP
[21]→ 50 NCPB 47 48 37 49
[22]→ 51 NECDYAD 48 49 47 37 36,
                38 39 0 1 1
```

[23] → NECOUT 51

```
[24]→ 52 DESINP 51
[25]→ 52 DPTICA 52 49 39 38 36 47 48 37
[26]→ 52 DLKURT 52 36 38 48 49
[27]→ 52 DLKSIZE 52 36 38 47 48
[28]→ 52 DESDYAD 52 48 49 47 37 36,
38 39 0 1, 16
[29]→ DESOUT 52 ∇
```

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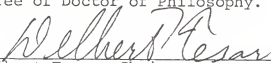
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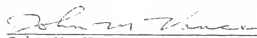
BIOGRAPHICAL SKETCH

Kim L. Spitznagel was born July 20, 1952, in Charleston, West Virginia. He attended primary school in Camp Hill, New Jersey, and York, Pennsylvania. In 1969 he began studies in mechanical engineering at Lehigh University and graduated with a Bachelor of Science in Mechanical Engineering in June, 1973. In September, 1973, he began attending the University of Florida and received a Master of Science in Engineering in December, 1975, and a Bachelor of Science in Computer and Information Science in March, 1978. He is a member of Pi Tau Sigma, Tau Beta Pi and Phi Kappa Phi honorary fraternities.

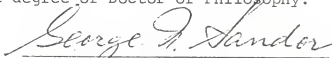
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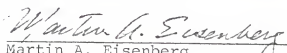
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